A R I T H M E T I C,

RATIONAL and PRACTICAL.

W H E R E I N

The properties of NUMBERS are clearly pointed out, the THEORY of the ſcience deduced from firſt principles, the methods of OPERATION demonſtratively explained, and the whole reduced to PRACTICE, in a great variety of uſeful RULES.

Conſiſting of THREE PARTS, viz.

I. VULGAR ARITHMETIC.
II. DECIMAL ARITHMETIC.
III. PRACTICAL ARITHMETIC.

By JOHN MAIR, A. M.

P A R T III.

PRACTICAL ARITHMETIC.

The SECOND EDITION.

E D I N B U R G H:
Printed for A. KINCAID & W. CREECH, and J. BELL.
MDCCLXXII.

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ARITHMETIC,

RATIONAL and PRACTICAL.

PART III.

PRACTICAL ARITHMETIC; or, Arithmetic Vulgar and Decimal, reduced to PRACTICE, in a great variety of ufeful rules.

CHAP. I.

FELLOWSHIP.

FELLOWSHIP, called alfo *Company*, or *Partnerfhip*, is when two or more perfons join their ftocks, and trade together, dividing the gain or lofs proportionally among the partners.

Fellowfhip is either without or with time, called alfo *fingle* or *double*.

I. *Fellowfhip without time.*

Queftions in fellowfhip without time are wrought by the following proportion.

As the total ftock
To the total gain or lofs,
So each man's particular ftock
To his fhare of the gain or lofs.

Queſt. 1. A and B make a joint ſtock: A puts in 12 l. and B 8 l.; they gain 5 l.: What is each man's ſhare?

	L.
A's ſtock	12
B's ſtock	8
Tot. ſtock	20

A. *Stock. Gain. Stock.*
If 20 : 5 : : 12
5
2|0)6|0
A's gain 3 l.

Stock. Gain. Stock.
B. If 20 : 5 : : 8
8
2|0)4|0
B's gain 2 l.

	L.	
A's gain	3	
B's gain	2	
Total gain	5	proof.

Queſt. 2. A, B, and C, make a joint ſtock: A puts in 78 l. B 117 l. and C 234 l.; they gain 265 l.: What is each man's ſhare?

Stock

	L.
Stock of A	78
B	117
C	234
Total ſtock	429

Stock Gain. Stock.
A. If 429 : 265 :: 78

```
              78
            ----
            2120
           1855
          ------
 429)20670(48 l.
     1716
     ----
      3510
      3432
      ----
        78
         20
       -----
   429)1560(3 s.
       1287
       ----
        273
         12
       -----
   429)3276(7 d.
       3003
       ----
        273
          4
       -----
   429)1092(2 f.
        858
       ----
       (234)
```

Stock. Gain. Stock.
B. If 429 : 265 :: 117

```
       117
     -----
      1855
      265
     265
     -----
429) 31005(72 l.
     3003 .
     -----
       975
       858
     -----
       117
        20
     -----
429) 2340(5 s.
     2145
     -----
      195
       12
     -----
429) 2340(5 d.
     2145
     -----
      195
        4
     -----
429) 780(1 f.
     429
     -----
     (351)
```

Stock. Gain. Stock.
C. If 429 : 265 :: 234

```
       234
     -----
      1060
      795
     530
     -----
429)62010(144 l.
    429 ..
    -----
    1911
    1716
    -----
     1950
     1716
     -----
      234
       20
     -----
429)4680(10 s.
    429
    -----
     390
      12
    -----
429)4680(10 d.
    429
    -----
     390
       4
    -----
429)1560(3 f.
    1287
    -----
    (273)
```

		L.	s.	d.	f.	Rem.
Anſ. Gain of	A	48	3	7	2	234
	B	72	5	5	1	351
	C	144	10	10	3	273
Proof		265	0	0	0	858

Note 1. When in any queſtion there happen to be remainders, they muſt be reduced equally low, ſo as to be all of one name; and then their ſum will be either equal

equal to the divisor, or exactly double, triple, &c. of it: and accordingly 1, 2, 3, &c. carried from the sum of the remainders, and added to the particular gains, will make up the total gain; or the divisor will always divide the sum of the remainders exactly, and the quot added to the particular gains will give the total gain.

Note 2. When the partners have equal shares of stock or capital, their shares of gain, loss, or neat proceeds, is found readily by dividing the total gain, loss, &c. by the number of partners, thus.

Suppose A, B, and C, were equally concerned in a voyage to Virginia, and that the neat proceeds turned out to 575 l. each partner's share will be L. 191 : 13 : 4, cast up as under.

	L.	s.	d.
3)575(	191	13	4

Note 3. When the fractions denoting the partners shares of stock or capital have all the same denominator, each partner's share of gain, loss, &c. is quickly found, by dividing the total gain, &c. by the denominator, and multiplying the quot by the respective numerators; thus.

Suppose A, B, C, and D, buy a ship for a sum of money, whereof A, B, and C, pay $\frac{1}{6}$ each, and D $\frac{3}{6}$ or $\frac{1}{2}$; and that her freight on a voyage amounts to 260 l. the partners shares are found as follows.

6)260

L.	43	6	8	to A
	43	6	8	to B
	43	6	8	to C
	130	0	0	to D
Proof	260			

Note 4. The operation may sometimes be shortened or facilitated by first finding the gain, loss, or neat proceeds, *per cent.* or *per* pound, and then working by the rules of practice, thus.

Suppose A, B, and C, join in an adventure to Barbadoes,

badoes, which, with all charges, amounted to 2000 l.; whereof A paid 400 l. B 600 l. and C 1000 l.; they have returns in goods, which being disposed of, the neat proceeds amounted to L. 4333 : 6 : 8 : What should each partner get?

If 2000 :	4333	6	8	:: 100
20 :	4333	6	8	:: 1
10 :	216|6	13	4	:: 1
	20			
	13|3			
	12			
	4|0			

216 13 4 *per cent.*
4

	L.	s.	d.
400 =	866	13	4
200 =	433	6	8
600 =	1300		
1000 =	2166	13	4

	L.	s.	d.
A gets	866	13	4
B gets	1300		
C gets	2166	13	4
Proof	4333	6	8

Again, suppose A, B, and C, join in an adventure to Jamaica, and ship off, on their joint credit, goods to the value of 3000 l.; and, to complete the cargo, the partners, from their own warehouses, ship off such goods as they had proper for the voyage, viz. A goods to the value of 100 l. B 200 l. C 300 l; the neat proceeds of their returns in sugar and rum amounted to 6930 l.: What share of this belongs to each partner?

3600 : 6930 :: 1
360 : 693 :: 1
120 : 231 :: 1
40 : 77 :: 1 : $\frac{77}{40}$ *per l.*

				Shares. L.	s.
A's stock	1100 × 77 =	84700, ÷ 40 =		2117	10
B's stock	1200 × 77 =	92400, ÷ 40 =		2310	
C's stock	1300 × 77 =	100100, ÷ 40 =		2502	10
			Proof	6930	

Decimally.

Decimally.

Quest. 3. A, B, and C, make a joint ſtock of 735 l. whereof

	L.	*s.*	*d.*		*L.*
A puts in	277	13	4	=	$277.\cancel{6}$
B ———	163	6	8	=	$163.\cancel{3}$
C ———	294			=	294.
Total ſtock	735				735.0

They gain 49 l.; required each man's ſhare.

$$\begin{array}{llllll} & L. & L. & & L. & L. & \\ \text{If} & 735 & :49 :: & \left\{ \begin{array}{l} 277.\cancel{6} \\ 163.\cancel{3} \\ 294. \end{array} \right. & \begin{array}{l} : 18.5\cancel{1} \\ : 10.\cancel{8}8 \\ : \underline{19.6} \\ \ \ 49.00 \end{array} & \begin{array}{l} \text{A} \\ \text{B} \\ \text{C} \\ \text{proof.} \end{array} \end{array}$$

But the eaſieſt way is to find the gain or loſs *per* l.; and by this multiply each man's ſtock. This common multiplier is found by the rule of three, or ſimply by dividing the total gain or loſs by the ſum of the ſtocks; thus:

$$\begin{array}{lllll} & L. & L. & L. & L. \\ \text{As} & 735 & : 49 & :: 1 & : .0\cancel{6} \text{ common multiplier.} \end{array}$$

$$\text{And} \left\{ \begin{array}{l} 277.\cancel{6} \times .0\cancel{6} = 18.5\cancel{1} \\ 163.\cancel{3} \times .0\cancel{6} = 10.\cancel{8}8 \\ 294. \ \times .0\cancel{6} = \underline{19.6} \\ \qquad\qquad\quad 49\ 00 \text{ proof.} \end{array} \right. \quad \begin{array}{llll} & L. & s. & d. \\ = & 18 & 10 & 2.\cancel{6} \\ = & 10 & 17 & 9.\cancel{3} \\ = & 19 & 12 & 0 \\ & 49 & 00 & 0.0 \end{array}$$

Again, ſuppoſe A put in 15 l. B 24 l. and C 33 l. and gain 63 l.; what is each man's ſhare of ſaid gain?

A 15 l.
B 24
C 33

72)63.0(.875 common multiplier.

		L.	*s.*	*d.*	
15 × .875 = 13.125	=	13	2	6	A's ſhare.
24 × .875 = 21	=	21	0	0	B's ſhare.
33 × .875 = 28.875	=	28	17	6	C's ſhare.
		63			proof.

MORE

MORE EXAMPLES.

Quest. 4. A, B, and C, make a joint ſtock: A puts in 460 l. B 510 l. and C 480 l.; they gain 340 l.: What is each partner's ſhare?

		L.	s.	d.	f.	Rem.
Anſ. Gain of	A	107	17	2	3	85
	B	119	11	8	2	110
	C	112	11	0	1	95
Proof.		340	00	0	0	290

Queſt. 5. Three perſons trade together: A puts in 230 l. B. 529 l. and C 344 l. 10 s.; they gain 520 l.; What is that to each?

		L.	s.	d.	Rem.
Anſ. Gain of	A	108	7	$7\frac{3}{4}$	271
	B	249	5	7	844
	C	162	6	9	1092

Queſt. 6. A, B, and C, make a joint ſtock of 3256 l.; whereof A puts in 1026 l. B 985 l. and C the reſt; by misfortune they loſe 2000 l.: What part of that muſt each bear?

		L.	s.	d.	Rem.
Anſ.	A	630	4	5	928
	B	605	0	$8\frac{3}{4}$	1240
	C	764	14	10	1088

Queſt. 7. Four partners, A, B, C, and D, built a ſhip, which coſt 1730 l.; and the freight for her firſt voyage amounted to 370 l.; of which A's ſhare was 74 l. B's 111 l. C's 148 l. and D's 37 l.: What was each partner's ſtock?

		L.
Anſ.	A's ſtock was	346
	B's - - -	519
	C's - -	692
	D's - - -	173
	Total ſtock	1730 proof.

Queſt. 8. A, B, and C, in company, gain 36 l.: A put in 20 l. B 30 l. C a ſum unknown; C took up 16 l. of the gain: What did A and B gain? and what did C put in?

Anſ.

		L.
Anſ.	A's gain	8
	B's gain	12
	C's ſtock	40

Queſt. 9. A, B, C, and D, in partnerſhip, had a joint ſtock of 600 l. and gained a certain ſum; of which A, B, and C, took up 60 l.; B, C, and D, 90; A, C, and D, 80; A, B, and D, 70: What was the ſtock and gain of each partner?

		Stock. L.	Gain. L.
Anſ.	A's	60	10
	B's	120	20
	C's	180	30
	D's	240	40

II. *Fellowſhip with time*

In fellowſhip with time, the gain or loſs is divided among the partners, both in proportion to the ſtocks themſelves, and alſo in proportion to the times of their continuance in company: for the ſame ſtock continued a double time, procures a double ſhare of gain; and continued a triple time, procures a triple ſhare of gain; that is, the ſhares of gain or loſs are as the products of the ſeveral ſtocks multiplied into their reſpective times: and accordingly queſtions belonging to this rule are wrought by the following proportion.

As the ſum of the products of the ſeveral ſtocks into their reſpective times
To the total gain or loſs,
So the product of each man's ſtock into his time
To his ſhare of the gain or loſs.

Queſt. 1. A put into company 40 l. for 3 months, B 75 l. for 4 months; they gain 70 l.: What ſhare muſt each man have?

A 40 × 3 = 120 third term for A's ſhare.
B 75 × 4 = 300 third term for B's ſhare.

420 firſt term.

A. If 420 : 70 :: 120 (L.)
120
42|0)840|0(20 l.
84.
(0)

B. If 420 : 70 :: 300 (L.)
300
42|0)2100|0(50 l.
210
(0)

	L.	
A's gain	20	
B's gain	50	
Total gain	70	proof.

Queſt. 2. A put into company 560 l. for 8 months, B 279 l. for 10 months, and C 735 l. for 6 months; they gained 1000 l.: What ſhare of the gain muſt each have?

A 560 × 8 = 4480 third term for A's ſhare.
B 279 × 10 = 2790 third term for B's ſhare.
C 735 × 6 = 4410 third term for C's ſhare.

11680 firſt term.

	L.		L.	s.	d.	f.	Rem.
A. If 11680 :	1000 ::	4480 :	383	11	2	3	208
B. If 11680 :	1000 ::	2790 :	238	17	4	3	80
C. If 11680 :	1000 ::	4410 :	377	11	4	1	880
		Proof	1000	00	0	0	1168

Decimally.

Queſt. 3. A, B, and C, enter into partnerſhip, viz.

	L.	s.	d.	
A puts in	58	10	0	for 8 months 3 weeks.
B ———	65	15	0	for 6 months 4 days.
C ———	48	17	6	for 12 months 2 weeks.

They

They gain 125 l. 14 s.: What is each man's ſhare?

	L.	*Mon.*	*Prod.*
A's ſtock	58.5 ×	8.75	= 511.875
B's ſtock	65.75 ×	6.1428	= 403.8891
C's ſtock	48.875 ×	12.5	= 610.9375
		Sum of the products	1526.7016

$$\text{As } 1526.7016 : 125.7 :: \begin{cases} 511.875 & : 42.1449 \text{ A} \\ 403.8891 & : 33.2539 \text{ B} \\ 610.9375 & : 50.3011 \text{ C} \end{cases}$$

Total gain nearly 125.6999 proof.

But here too the eaſieſt method is to find the gain or loſs per l. and then make this a common multiplier of the ſeveral products of the ſtocks and times. Thus,

As 1526.7016 *L.* : 125.7 *L.* :: 1 : .08233 common multiplier.

	A	B	C
	511.875	403.8891	610.9375
Multiplier 33280. inverted	33280.	33280.	33280.
	409500	323111	488750
	10237	8077	12218
	1535	1211	1832
	153	121	183
	42.1425	33.2520	50.2983

The ſhares of the gain are nearly the ſame as before.

MORE EXAMPLES.

Queſt. 4. A, B, and C, hire a paſture for 24 l.: A puts in 40 cows for 4 months, B 30 cows for 2 months, and C 36 cows for 5 months: What ſhare of the rent muſt each pay?

		L.	s.
Anſ.	A's ſhare of rent	9	12
	B's ſhare -	3	12
	C's ſhare -	10	16
	Proof	24	

Queſt. 5. A, B, and C, agreeing to trade together, A puts in 529 l. for 4 months, B 329 l. for 7 months, and C 900 l. for 2 months; they gain 540 l.: What is each man's ſhare?

		L.	s.	d.	Rem.
Anſ.	A	183	14	8	2304
	B	199	19	5	1332
	C	156	5	$10\frac{3}{4}$	2583

Queſt. 6. A and B enter into partnerſhip for a year; accordingly A put in the firſt of January 50 l.; but B could not put any money in till the firſt of May: What ſum muſt B then put in to be intitled to an equal ſhare of the gain at the year's end? *Anſ.* 75 l.

Queſt. 7. A, B, and C, agree to trade in company for 12 months: A put in at firſt 364 l. and at the end of 4 months he put in 40 l. more; B put in at firſt 408 l. and at the end of 7 months he took out 86 l.; C put in at firſt 148 l. and at the end of 3 months he put in 86 l. more, and at the end of other 5 months he put in 100 l. more; at the year's end their gain was 1436 l.: What is each man's ſhare?

		L.	s.	d.	Rem.
Anſ.	A's gain	556	3	6	6192
	B's gain	529	16	9	5496
	C's gain	349	19	8	416

Note. Queſtions in double fellowſhip are proper exerciſes for the learner, but ſeldom occur in real buſineſs; differences in point of time being uſually adjuſted by an intereſt-account.

CHAP.

CHAP. II.

FACTORAGE.

FActors are agents or doers, reſiding abroad, or beyond ſeas, who act for their employers; and for their trouble are generally allowed, in name of commiſſion or fees, ſo much *per cent.* The computations they have occaſion for are ſuch as occur in the ſolution of the following or like queſtions.

Queſt. 1. A B's ſales per the Swan amount to 675 l. 18 s. 8 d.: What is the commiſſion at 8 *per cent.?*

		L.	*s.*	*d.*		*Decimally.*	*L.*
If	100 : 8 : :	675	18	8	If	100 : 8 : :	675.93
				8			8
	L.	54.07	9	4		L.	54.0746
		20					20
	s.	1.49				s.	1.493
		12					12
	d.	5.92				d.	5.920
		4					4
	f.	3.68				f.	3.68

Queſt. 2. A factor buys, and ſhips off to his employer, goods to the value of L. 1334 : 11 : $5\frac{1}{4}$: What is the commiſſion at 5 *per cent.?*

If

	L.	s.	d.
If 100 : 5 ::	1334	11	$5\frac{1}{4}$
			5
L.	66.72	17	$2\frac{1}{4}$
	20		
s.	14.57		
	12		
d.	6.86		
	4		
f.	3.45		

Decimally.

	L.
If 100 : 5 ::	1334.571875
	5
L.	66.72859375
	20
s.	14.57187500
	12
d.	6.862500
	4
f.	3.4500

Quest. 3. The prime coſt of a parcel of conſigned goods, per invoice, is L. 37 : 8 : $1\frac{1}{2}$; theſe the factor ſells at 75 *per cent.* advance on the invoice: What is their value in the ſale?

	L.	s.	d.
If 100 : 75 ::	37	8	$1\frac{1}{2}$
	20		
	748		
	12		
	8977		
	4		
	35910		
	75		
	17955		
	25137		
4)	26932.50		
12)	6733		
20)	56|1		
	28	1	1 advance.
	37	8	$1\frac{1}{2}$ prime coſt.
	65	9	$2\frac{1}{2}$ value in the ſale.

Decimally.

	L.
If 100 : 175 ::	37.40625
	175
	18703125
	26184375
	3740625
L.	65.4609375
	20
s.	9.2187500
	12
d.	2.62500
	4
f.	2.500

Quest.

Quest. 4. The neat proceeds of a sale is 620 l. 12 s. $6\frac{1}{2}$ d.: To what value ought the factor to make returns, so as to clear the debt, and have 5 *per cent.* commission?

```
If 105 : 100 : : 620    12   6½
                   20
              -------
                12412
                   12
              -------
               148950
                    4
              -------
               595802
                   100
              ---------   4)
        105)59580200(567430
            525.....   -------
            ----    12)141857   2 f.
             708       -------
             630   2|0)1182|1   5 d.
             ---       -------
              780       591     1 s.
              735
              ---
               452
               420
               ---
                320
                315
                ---
                (50)
```

Decimally,

Decimally.

L.

If 105 : 100 : : $620.62708\dot{3}$

$105)62062.7083(591.0734$

525			
956		s.	1.4680
945			12
112		d.	5.616
105			4
770		f.	2.464
735			
358			
315			
433			
420			
(13)			

	L.	*s.*	*d.*
Value of returns	591	1	$5\frac{1}{2}$
Commiſſion	29	11	1
Proof	620	12	$6\frac{1}{2}$

Queſt. 5. A factor buys for his employer goods to the value of L. 7864 : 16 : 8 : What will his commiſſion come to at $2\frac{1}{2}$ *per cent.* ?

$2\frac{1}{2}$

$2\frac{1}{2}$ *per cent.* is $\frac{1}{40}$ of 100; and therefore,

By Practice,

	L.	s.	d.	L.	s.	d.	
4|0)786|4		16	8(	196	12	5	*Anf.*

Rem. 24
20
49|6(
Rem. 16
12
20|0(

Decimally.

If 100 : 2.5 : : L. 7864.8$\dot{3}$
2.5
393241$\dot{6}$
1572966$\dot{6}$
L. 196.6208$\dot{3}$
20
s. 12.416$\dot{6}$
12
d. 5.000

Queſt. 6. If I grant a draught on London for 4835 l. and be allowed $1\frac{3}{8}$ *per cent.* in name of premium or commiſſion, what will that amount to?

		L.
1 *per cent.* $= \frac{1}{100}$	=	48.35
$\frac{2}{8} = \frac{1}{4}$ of that	=	12.0875
$\frac{1}{8} = \frac{1}{2}$ of that	=	6.04375
	L.	66.48125
		20
	s.	9.6250
		12
	d.	7.500

Anſ. L. 66 : 9 : $7\frac{1}{2}$

Some, inſtead of paying a premium, chuſe to take the bill payable ſo many days after date or ſight; and in this way, 73 days are reckoned equal to 1 *per cent.*

Queſt. 7. A factor ſhips off, by his employer's order, goods to the value of L. 97 : 5 : 8 : What will his commiſſion at 2 *per cent.* amount to?

2 *per cent.* = $\frac{1}{50}$ of 100.

By Practice,

```
5|0)9|7  5  8(1  18  10¾ Anſ.
Rem. 47
     20
    ----
    94|5(
    ----
Rem. 45
     12
    ----
    54|8(
    ----
Rem. 48
      4
    ----
    19|2(
    ----
    (42)
```

Decimally.

```
                 L.
If 100 : 2 :: 97.283
                   2
             -------
          L. 1.94566
                  20
             -------
           s. 18.913
                 12
             -------
           d. 10.960
                  4
             -------
            f. 3.84
```

MORE EXAMPLES.

Queſt. 8. A factor ſhips off for his employer goods to the value of L. 1629 : 4 : 3½ : What will his commiſſion at 5 *per cent.* amount to? *Anſ.* L. 81 : 9 : 2½.

Queſt. 9. A factor owes his employer L. 262 : 11 : 7¾: How much ought he to remit, in order to clear the debt, and have 2½ *per cent.* commiſſion on the ſum remitted? *Anſ.* L. 256 : 3 : 6¾.

Queſt. 10. A factor negotiates a bill of L. 76 : 10 : 9: What commiſſion may he charge on this head at 1½ *per cent?* *Anſ.* L. 1 : 2 : 11½.

CHAP.

CHAP. III.

TARE and TRET, &c.

GRoſs *weight* is the weight both of the commodity, and of that which contains it, whether caſk, hogſhead, barrel, bag, ſack, or wrapper.

Tare is an allowance on weighable goods, made by the King to the importer, or by the ſeller to the buyer, in conſideration of the outſide package; ſuch as, caſk, bag, box, cheſt, wrappers, &c.

Tret is an allowance of 4 lb. on 104 lb. on garbled goods, or ſuch as are ſold by the pound-weight, granted for break, waſte, or duſt mixed with the goods.

Cloff or *clough*, called alſo *draught*, is another ſmall allowance on ſome weighable goods, uſually 2 lb. on every 3 C. to turn the ſcale, or make the weight hold out, when the goods are re-weighed. This is claimed chiefly, or only, by the citizens of London.

Suttle, or *ſuttle weight*, is what remains after the tare is deduced from the groſs: and is ſo called only when tret is allowed; for when there is no tret, there is no ſuttle.

Neat weight is the weight of the goods after all given allowances are deduced. Thus, if tare, tret, and cloff, be allowed, the neat weight is what remains after theſe three allowances are deduced. If tare and tret only be allowed, the neat weight is what remains after theſe two allowances are deduced. If tare only be allowed, the neat weight is what remains after that allowance is deduced.

I. *Tare.*

Tare admits of three varieties: for ſometimes the hogſhead, caſk, or barrel, is weighed before the goods are packed, and the tare inſerted in the invoice along with the groſs weight; or the tare is ſo much on the whole. Again, the allowance for tare is ſometimes ſo

much per frail, per firkin, per hogſhead, per cheſt, per bag, &c. Laſtly, the allowance for tare is ſometimes ſo much per C.

1. When the tare is inſerted in the invoice, or is ſo much on the whole, the neat weight is found by ſubtracting the tare from the groſs.

EXAMPLE I.

Suppoſe a factor in Jamaica buys for the uſe of his employer in Britain, the three following hogſheads of ſugar, their neat weight in pounds, that being the uſual form, is computed as under.

N°	C.	Q.	lb.	*Tare in pounds.*
1.	16	1	14	114
2.	14	3	7	110
3.	15	2	21	116
	46	3	14	340

46
46
46
4698

5250 groſs.
340 tare.

lb. 4910 neat.

EXAMPLE II.

A grocer buys 54 cheſts of ſpice, each cheſt weighing 4 C. 2 Q. 14 lb. groſs; tare on the whole, 21 C. 3 Q. 18 lb.: Required the neat weight.

C.	Q.	lb.	
4	2	14	$54 = 9 \times 6$.
		9	
41	2	14	
		6	
249	3	0 total groſs.	
21	3	18 tare.	
Anſ. 227	3	10 neat.	

2. When

2. When the tare is so much per frail, per firkin, per chest, per bag, &c. multiply the allowance for tare by the number of frails, firkins, chests, or bags; subtract the product from the gross; and the remainder is the neat weight.

Or, from the gross weight of 1 frail, firkin, hogshead, &c. subtract its tare; the remainder is the neat weight of 1 frail, firkin, &c.; which, multiplied by the number of frails, &c. gives the total neat weight.

EXAMPLE I.

What is the neat weight of 7 frails of raisins, each weighing 3 C. 3 Q. 10 lb. gross, tare at 24 lb. per frail?

	C.	Q.	lb.			lb.	C.	Q.
	3	3	10			24		
			7	frails.	Frails	7		
	26	3	14	gross.		168 =	1	2
	1	2	0	tare				
Ans.	25	1	14	neat.				

Or,

C.	Q.	lb.	
3	3	10	gross of 1 frail.
		24	tare.
3	2	14	neat of 1 frail.
		7	
25	1	14	neat of 7 frails.

EXAMPLE II.

What is the neat weight of 14 bags of cotton, each weighing 2 C. 2 Q. 7 lb. tare at 9 lb. per bag?

C.

	C.	Q.	lb.
	2	2	7
14 = 7 × 2			7
Bags.	17	3	21
14			2
9	35	3	14
126	35		
	35		
	3598		
	4018 grofs.		
	126 tare.		
	lb. 3892 neat.		

Or,

C.	Q.	lb.	
2	2	7	grofs of 1 bag.
		9	tare.
2	1	26	neat of 1 bag.
		7	
17	1	14	
		2	
34	3	00	neat of 14 bags.
34			
34			
3484			
3892 lb. neat.			

Note. The tare for feveral forts of goods is afcertained in the book of rates. Thus, for every bale of filk from Smyrna or Cyprus, weighing 300 lb. or upwards, the tare is fixed at 16 lb.; for every bale weighing 200 lb. or upwards to 300, the tare is 14 lb.; and for every bale under 200 lb. the tare is 12 lb.

Accordingly the neat weight of the five following bales of filk from Smyrna, will be made out from the book of rates, as under.

N°	lb.	*Tare.*
1.	346	16
2.	300	16
3.	284	14
4.	200	14
5.	158	12
Grofs	1288	72
Tare	72	
Neat	1216	

3. When

3. When the tare is ſo much per C. work by one or other of the two rules following.

RULE I.

If the tare be 14 or 16 per C. take accordingly $\frac{1}{8}$ or $\frac{1}{7}$ of the groſs for the tare; which, ſubtracted from the groſs, leaves the neat weight.

EXAMPLE I.

lb.		C.	Q.	lb.		
		493	2	24	groſs.	Tare 14 per C. lb.
14	$\frac{1}{8}$	61	2	24	tare.	
		432	0	0	neat.	

EXAMPLE II.

lb.		C.	Q.	lb.		
		119	3	0	groſs.	Tare 16 per C. lb.
16	$\frac{1}{7}$	17	0	12	tare.	
		102	2	16	neat.	

RULE II.

If the tare be any other than 14 or 16 per C. firſt work as above for 14 or 16, and then take aliquot parts of the quot.

EXAMPLE I.

lb.		C.	Q.	lb.		
		57	2	24	groſs.	Tare 7 per C. lb.
14	$\frac{1}{8}$	7	0	24		
7	$\frac{1}{2}$ of $\frac{1}{8}$	3	2	12	tare.	
		54	0	12	neat.	

EX-

EXAMPLE II.

lb.		C.	Q.	lb.		lb.
		573	2	13	grofs.	Tare, 17 per C.
14	$\frac{1}{8}$	71	2	$22\frac{1}{2}$		
2	$\frac{1}{7}$ of $\frac{1}{8}$	10	0	27		
1	$\frac{1}{2}$ of $\frac{1}{7}$ of $\frac{1}{8}$	5	0	$13\frac{1}{2}$		
17		87	0	7	tare.	
		486	2	6	neat.	

EXAMPLE III.

lb.		C.	Q.	lb.		lb.
		44	2	12	grofs.	Tare 21 per C.
14	$\frac{1}{8}$	5	2	$8\frac{1}{2}$		
7	$\frac{1}{2}$ of $\frac{1}{8}$	2	3	$4\frac{1}{4}$		
21		8	1	$12\frac{3}{4}$	tare.	
		36	0	$27\frac{1}{4}$	neat.	

In the examples following, the firft part of the compound fraction only is expreffed, the other part or parts, for brevity's fake, being omitted.

EXAMPLE IV.

lb.		C.	Q.	lb.		lb.
		170	2	14	grofs.	Tare 6 per C.
16	$\frac{1}{7}$	24	1	14		
8	$\frac{1}{2}$	12	0	21		
2	$\frac{1}{4}$	3	0	$5\frac{1}{4}$		
6		9	0	$15\frac{3}{4}$	tare.	
		161	1	$26\frac{1}{4}$	neat.	

EX-

EXAMPLE V.

lb.		C.	Q.	lb.		lb.
		550	1	0	grofs.	Tare 10 per C.
16	$\frac{1}{7}$	78	2	12		
8	$\frac{1}{2}$	39	1	6		
2	$\frac{1}{4}$	9	3	$8\frac{1}{2}$		
10		49	0	$14\frac{1}{2}$	tare.	
		501	0	$13\frac{1}{2}$	neat.	

EXAMPLE VI.

lb.		C.	Q.	lb.		lb.
		237	1	14	grofs.	Tare 12 per C.
16	$\frac{1}{7}$	33	3	18		
8	$\frac{1}{2}$	16	3	23		
4	$\frac{1}{2}$	8	1	$25\frac{1}{2}$		
12		25	1	$20\frac{1}{2}$	tare.	
		211	3	$21\frac{1}{2}$	neat.	

Note 1. That 14 and 16 pounds per C. may be confidered as the ftandards of tare, in regard tare at any other rate may eafily be computed from them, as is done in the above examples.

Note 2. It is not ufual in computing tare to take notice of any thing lower, or lefs, than $\frac{1}{4}$ lb.; and accordingly, in Ex. 2. fome fmall fractions are neglected or thrown away.

Note 3. Though the method of computing tare, explained above, be the more ufual way; yet fome chufe to multiply the pounds grofs by the rate, or allowance of tare, and dividing the product by 112, the quot gives the tare in pounds. Ex. 1. wrought in this manner follows,

C.	Q.	lb.		lb.
57	2	24 gross.		Tare 7 per C.

```
       57
      57
     5780
     ----
     6464
        7 lb.     C.  Q. lb.
    ------
112)45248(404 = 3   2  12  as before.
    448
    ----
      448
      448
      ---
      (0)
```

Others multiply the C.'s by the rate of tare, and for the quarter and pounds take proportional parts, thus.

	C.	Q.	lb.		lb.
	57	2	24 gross.		Tare 7 per C.
	7				
Tare of 57 C. is	399				
—— of 2 Q. is	$3\frac{1}{2}$				
—— of 16 lb. is	1				
—— of 8 lb. is	$0\frac{1}{2}$	C.	Q.	lb	
Total tare	404 =	3	2	12	as above.

Note 4. The tare for oil in uncertain casks is 18 lb. per C. but pays duty by the gallon, and the number of gallons is computed from the neat weight, by allowing $7\frac{1}{2}$ lb. to a gallon; that is, multiply the neat pounds by 2, the product divided by 15, quotes gallons; or, divide the product by 3, and that quot by 5.

EXAMPLE.

		C. Q. lb.	
		128 2 0	oil grofs. Tare 18 lb. per C.
		1536	
		56	
		14392	lb. grofs.
lb.			
16	$\frac{1}{7}$	2056	
2	$\frac{1}{8}$	257	
		2313	lb. tare.
		12079	lb. neat.
		2	
		24158	half-pounds.
	$\frac{1}{15}$	$1610\frac{1}{2}$	gallons.

In working decimally, the tare may be found by multiplying the grofs weight by the number of pounds tare in one C. or by the decimal of the tare; and this product fubtracted from the grofs weight leaves the neat. But it is eafier, and more ufual, to work by one or other of the three methods following.

1. Multiply the grofs weight by the neat pounds in one C. and the product is the pounds neat.

	C.	Q.	lb.	
Ex.	573	2	13	grofs. Tare 17 lb. per C.

C.
573.616 grofs.
95 lb. neat in 1 C.

2868080
5162544

	C.	Q.	lb.	
112)54493.520(	486	2	$5\frac{1}{2}$	neat.

448 . .

969
896

733
672

(61)

2. Multiply the grofs weight by the decimal of the neat pounds in 1 C. and the product is neat C.

lb.	C.
95	= .8482

C.
Ex. 573.616 grofs.
2848. multiplier inverted.

4588928
229446
45889
1147

486.5410 neat weight nearly.

3. Work by aliquot parts, as in Practice.

C.
Ex. 573.616 grofs. Tare 17 lb. p. C.

lb.		
14	$= \frac{1}{8}$ of the grofs	= 71.702
2	$= \frac{1}{7}$ of the former	= 10.243
1	$= \frac{1}{2}$ of the former	= 5.121
17		87.066 tare.

486.550 neat weight nearly.

In

In working by aliquot parts it will be convenient to have at hand a table of diviſors, ſuch as the following; which is conſtructed in the ſame manner as the table of rates and diviſors in decimal practice.

lb. Tare.	*Diviſors.*	*lb. T.*	*Diviſors.*	*lb. T.*	*Diviſors.*
1	2, 7, 8	10	2, 7, + 4	19	7, + 4, — 20
2	7, 8	11	8, 7, — 12	20	7, + 4
3	7, 8, + 2	12	8, — 7	21	8, + 2
4	4, 7	13	8, — 7, + 12	22	7, + 4, + 2
5	4, 7, + 4	14	8	23	7, + 2, — 8
6	4, 7, + 2	15	8, + 7, — 2	24	7, + 2
7	2, 8	16	7	25	7, + 2, + 8
8	2, 7	17	8, + 7, + 2	26	7, + 2, + 4
9	2, 7, + 8	18	7, + 8	27	7, + 2, + 4, + 2

II. *Tret.*

Tret being always 4 lb. per 104 lb. ſuttle, is found by taking $\frac{1}{26}$ of the ſuttle-weight.

EXAMPLE I.

	C.	*Q.*	*lb.*	
$\frac{1}{26}$	428	1	19	ſuttle. Tret 4 lb. per 104 lb.
	16	1	$25\frac{1}{2}$	tret.
	411	3	$21\frac{1}{2}$	neat.

```
     C.  Q. lb.  C.  Q.  lb.
26)428   1  19(16   1   25½
   26
   ---
   168
   156
   ---
    12 rem.
     4
    ---
 26)49(1
    26
    ---
    23 rem.
    28
   ---
   193
   47
   ---
26)663(25½
   52
   ---
   143
   130
   ---
   (13)
```

EXAMPLE II.

```
lb.|      |  C.   Q.  lb.
   |      | 836   2   17    grofs.   Tare 22 lb. per C.
   |      |-----------------          Tret 4 lb. per 104 lb.
16 | 1/7  | 119   2    2¼
 4 | 1/4  |  29   3   14½
 2 | 1/2  |  14   3   21¼
---|      |-----------------
22 |      | 164   1   10  tare.
   |      |-----------------
   | 1/26 | 672   1    7  futtle.
   |      |  25   3   12  tret.
   |      |-----------------
   |      | 646   1   23  neat.
```

Note. The neat weight may alfo be had by annexing two ciphers to the futtle pounds, and then dividing by 104. Ex. 1. wrought in this manner follows.

C.

```
C.    Q.  lb.
428   1   19 ſuttle.   Tret 4 lb. per 104 lb.
 5136
   47       lb.         C.   Q.  lb.
104)4798300(46137½=411  3   21½ as before.
    416····
    ---
     638
     624
     ---
      143
      104
      ---
       390
       312
       ---
        780
        728
        ---
        (52)
```

In working decimally, multiply the ſuttle by $.0385 = \frac{4}{104}$, and the product is the tret; which deduced from the ſuttle, leaves the neat.

But it is ſhorter to multiply the ſuttle by $.9615 = \frac{100}{104}$, and the product is the neat. Ex. 2 done in this manner follows.

```
C.    Q.  lb.   C.
836   2   17 = 836.6517 groſs.   Tare 22 lb. per C.
                                 Tret 4 lb. per 104 lb.
             ⎧ 119.5216
7 + 4 + 2    ⎨  29.8804
             ⎩  14.9402
               --------
               164.3422 tare.
               --------
               672.3095 ſuttle.
                   5169 multiplier inverted.
               --------
               60507855
                4033857
                  67230
                  33615
               --------
              646.42557 neat weight nearly.
```

III. *Cloff.*

III. *Cloff.*

When tret is allowed, the remainder, after deducing the tare from the groſs, is called *ſuttle*; and when cloff is allowed, the remainder of the ſuttle-weight, after deducing the tret, is again called *ſuttle*, or rather *ſubſuttle*.

Cloff, being always 2 lb. on every 3 C. ſubſuttle, is found by taking $\frac{1}{168}$ of the ſubſuttle weight.

Or, divide the ſubſuttle C.'s by 3, the quot is cloff in double pounds; theſe divided by 14 quot quarters; and the remainder is double pounds.

EXAMPLE I.

C.	*Q.*	*lb.*	
4033	2	14	ſubſuttle. Cloff 2 lb. per 3 C.
24	0	1	cloff.
4009	2	13	neat.

Method 1.

```
168)4033(24 C.
    336·
    ----
     673
     672
     ---
       1
       4
   ----
168)6(0 Q.
   28
   ---
168)182(1 lb.
    168
    ---
    (14)
```

Method 2.

```
3)4033(1
  -----
          4)  C.   Q.  lb.
14)1344(96 (24    0    1
   126·
   ----
    84
    84
   ----
    (0)
```

Note, The 1 C. remaining in the firſt diviſion, and the 2 Q. in the ſubſuttle, afford the 1 lb. cloff.

EX-

EXAMPLE II.

C.	Q.	lb.	
32	3	20	grofs.
4	0	13	tare.
28	3	7	futtle.
1	0	12	tret.
27	2	23	fubfuttle.
		$18\frac{1}{4}$	cloff.
27	2	$4\frac{3}{4}$	neat.

	lb.	
Tare	14	per C.
Tret	4	per 104.
Cloff	2	per 3 C.

	lb.
Note, The 27 C. afford	18 cloff.
And the 2 Q. afford -	$8\frac{1}{4}$
	$18\frac{1}{4}$

The cloff may be found decimally by multiplying the fubfuttle by .00595 $=\frac{1}{168}$.

Any operation in tare and tret may be proved by working the fame queftion by the rule of three.

PRACTICAL QUESTIONS.

1. What is the neat weight of 346 C. 3 Q. 12 lb. grofs, tare 12 lb. per C.? *Anf.* 309 C. 2 Q. $21\frac{1}{2}$ lb. neat.

2. What is the neat weight of 139 C. 1 Q. 22 lb. grofs, tare 8 lb. per C. tret 4 lb. per 104, and cloff 2 lb. per 3 C.? *Anf.* 123 C. 3 Q. $1\frac{3}{4}$ lb. neat.

3. The neat weight being 216 C. tare 14 per C. what is the tare and grofs? *Anf.* The tare is 30 C. 3 Q. 12 lb. and the grofs 246 C. 3 Q. 12 lb.

4. The neat weight being 97 C. 3 Q. 2 lb. tret 4 lb. per 104, and tare 16 lb. per C. what is the tret, futtle, tare, and grofs? *Anf.* The tret is 3 C. 3 Q. 18 lb. the futtle 101 C. 2 Q. 20 lb. the tare 16 C. 3 Q. 22 lb. and the grofs 118 C. 2 Q. 14 lb.

5. The grofs weight being 44 C. 3 Q. 18 lb. the tare on the whole 1 C. 1 Q. 2 lb. and the tret 4 lb. per

104, what is the neat weight, and what the price, at 2 l. 16 s. per C. neat? *Anſ.* The neat weight is 41 C. 3 Q. 24 lb. and the price is 117 l. 10 s.

6. Six hogſheads contain in all 55 C. 0 Q. 26 lb. groſs; now tare being allowed at 30 lb. per hogſhead, tret at 4 lb. per 104, and cloff at 2 lb. per 3 C. what will be the neat weight, and what the price thereof, at 9 d. per lb. neat? *Anſ.* Neat weight 51 C. 1 Q. $0\frac{3}{4}$ lb. price 215 l. 5 s. $6\frac{3}{4}$ d.

CHAP. IV.

BARTER.

BArter, or truck, is the exchanging of one commodity for another; in doing of which the price of one of the commodities, and an equivalent quantity of the other, muſt be found either by practice, or by the rule of three.

Queſt. 1. How many pounds of cotton, at 9 d. per lb. muſt be given in barter for 13 C. 3 Q. 14 lb. of pepper, at 2 l. 16 s. per C.?

Firſt, Find the price or value of the commodity whoſe quantity is given, as follows.

		C.	Q.	lb.		L.	s.
		13	3	14	at	2	16
2 l.							
16 s.		26					
2 Q.		10	8				
1 Q.		1	8				
14 lb.			14				
			7				
	L.	38	17				

Secondly,

Secondly, Find how much cotton, at 9 d. per lb. 38 l. 17 s. will purchase, as under.

```
    d.  lb.    L.  s.
If  9 : 1  : : 38  17
                   20
               ------
                  777
                   12
               ------
            9)9324(        C.  Q.
Ans.   1036 lb. = 9    1
```

If the above question be wrought decimally, the operation may stand as follows.

```
   C.   L.       C.
If 1  : 2.8 : : 13.875
                   2.8
                ------
                111000
                 27750     lb.     C.  Q.
                ------
     .0375)38.8500(1036 = 9    1  Ans.
           37 5...
           ------
             1350
             1125
             ------
              2250
              2250
             ------
```

Quest. 2. A merchant receives 126 yards of cloth in barter for 189 gallons brandy, at 6 s. 8 d. per gallon, what was the cloth valued at per yard?

```
   |   Gall.  s.  d.        yds.    L.      yd.   s.
   |   189 at 6   8      If 126  :  63  : :  1  :  10  Ans.
1/3| L. 63
```


The

The decimal operation of the same question follows.

$$\begin{array}{l} \quad G. \quad L. \quad 3)\,G. \\ \text{If } 1 : .3 :: 189 \\ \qquad\qquad \overline{}\; L. \quad s. \\ \qquad 126)63.0(.5 = 10 \; Ans. \\ \qquad\quad 630 \end{array}$$

Quest. 3. A and B barter; A gets 20 C. of cheese, at 21 s. 6 d. per C.; B gets eight pieces of cloth, at 3 l. 14 s. per piece: What is the balance, and to whom due?

	L.	s.
B's 8 pieces cloth, at 3 l. 14 s. come to	29	12
A's 20 C. cheese, at 21 s. 6 d. amount to	21	10
Balance due to B. - -	8	2

Quest. 4. A and B barter; A hath ginger worth 1 l. 17 s. 4 d. per C. ready money, but in barter he will have 2 l. 16 s. per C.; B hath nutmegs worth L. 5 12 s. per C. ready money: How must B rate his nutmegs in barter, to be on an equal footing with A? Say,

$$\begin{array}{l} \quad L.\ s.\ d. \quad L.\ s. \quad L.\ s \quad L.\ s. \\ \text{If } 1\ 17\ 4 : 2\ 16 :: 5\ 12 : 8\ 8\ Ans. \end{array}$$

The value or price of the goods received and delivered in barter being always equal, it is obvious, that the product of the quantities received and delivered, multiplied into their respective rates, will be equal.

Hence arises a rule which may be used with advantage in working several questions; namely, Multiply the given quantity and rate of the one commodity, and the product divided by the rate of the other commodity quotes the quantity sought; or divided by the quantity quotes the rate.

Quest. 5. How many yards of linen, at 4 s. per yard, should I have in barter for 120 yards of velvet, at 15 s. 6 d.?

$$\begin{array}{l} Yards. \quad Sixpences \qquad\qquad Sixp. \quad Yards. \\ 120 \times 31 = 3720, \text{ and } 8)3720(465\ Ans. \end{array}$$

Quest.

Quest. 6. If I receive 140 yards of broad cloth in barter, for 28 yards of velvet, at 16 s. 8 d. per yard, what was the broad cloth valued at per yard?

Yds. Groats. *s. d.*
28 × 50 = 1400, and 140)1400(10 groats, or 3 4 *Ans.*

Quest. 7. How many hats at 7 s. napkins at 4 s. and caps at 1 s. of each an equal number, may I have in barter for 280 yards of linen, at 3 s. per yard?

Ans. [sort.
280 × 3 = 840, and 7 + 4 + 1 = 12)840(70 of each

Quest. 8. A has 41 C. hops, at 30 s. per C. for which B gives him 20 l. in money, and the rest in prunes, at 5 d. per lb.: How many prunes did B give A beside the 20 l.? *Ans.* 17 C. 3 Q. 4 lb.

Quest. 9. A has 608 yards broad cloth, at 14 s. per yard; for which B gives him 125 l. 12 s. and 85 C. 2 Q. 24 lb. bees wax: How was the wax rated per C.? *Ans.* At 3 l. 10 s. per C.

Quest. 10. A hath 120 yards druggets, worth 6 s. per yard, but in barter he will have 8 s. per yard; B hath shalloon worth 4 s. per yard: How ought B to rate his shalloon in barter? and how many yards ought he to give A for his druggets? *Ans.* The shalloon is to be rated at 5 s. 4 d. per yard; and 180 yards to be given for the druggets.

Quest. 11. A hath 324 yards linen, at 4 s. 6 d. per yard; for which B gives him hats, at 7 s. per piece, and stockings at 6 s. 6 d. per pair: How many hats and pairs of stockings, of each an equal number, must B give A for his linen? *Ans.* 108 hats, and as many pairs of stockings.

Quest. 12. A has 100 pieces silk, worth 3 l. per piece; but barters them at 4 l. per piece, and takes their value from B in wool, at 7 l. 10 s. per C. which was only worth 6 l. per C.: How much wool must B give A for the silk? Whether was A or B the gainer? and how much? *Ans.* B gives A $53\frac{1}{3}$ C. wool, and A gains 20 l.

CHAP.

CHAP. V.

LOSS and GAIN.

QUeftions relating to lofs and gain, are likewife folved by practice, or the rule of three, as in the following examples.

Queft. 1. A merchant bought 436 yards of broad cloth, at 8 s. 6 d. per yard, which he fold at 10 s. 4 d. per yard: How much did he gain on the whole?

Find both the buying price and felling price by practice or the rule of three, and their difference is the gain.

	Yds.	*s.*	*d.*	
	436	at 8	6	
6 s.	130	16		
2 s. 6 d.	54	10		
	185	6		buying price.

	Yds.	*s.*	*d.*	
	436	at 10	4	
6 s.	130	16	0	
1 s.	21	16	0	
3 s. 4 d.	72	13	4	
	225	5	4	felling price.
	185	6	0	buying price.
	39	19	4	gain.

Or, Find the gain per yard, by fubtracting the buying rate from the felling rate; and then find the gain on 436 yards, by practice or by the rule of three.

s.	d.			Yds.
10	4	buying rate.	1 s.	436 at 1 s. 10 d.
8	6	selling rate.	6 d.	218
1	10	gain per yard.	4 d.	145 4
			$\frac{1}{20}$	79|9 4
				L. 39 19 4 gain.

Yd. d. Yds.
Or, If 1 : 22 :: 436
22
———
872
872
———
12)9592 d.
2|0)79|9 4.
Gain L. 39 19 4

Yd. L. Yds. L. s. d.
Or, decimally, If 1 : .091$\dot{6}$ :: 436 : 39 19 4

Quest. 2. A draper bought 124 yards of holland for 31 l.: How must he sell it per yard to gain 10 l. 6 s. 8 d. on the whole?

To the buying price add the intended gain, and then find the rate per yard, by the rule of three.

L.	s.	d.	
31	0	0	buying price.
10	6	8	gain.
41	6	8	selling price.

Yds. L. s. d. Yd. s. d.
If 124 : 41 6 8 :: 1 : 6 8 *Ans.*

Yds. L. Yd. s. d.
Or, decimally, If 124 : 41.$\dot{3}$ :: 1 : 6 8

Quest. 3. A grocer bought 3 C. 1 Q. 14 lb. of cloves, at 2 s. 4 d. per lb. which he sold for 52 lb. 14 s.: How much did he gain on the whole?

Find

Find the buying price, ſubtract it from the ſelling price, and the remainder is the gain, viz. 8 l. 12 s.

Queſt. 4. A parcel of goods was bought for 60 l. and ſold for 75 l.: What was gained *per cent.?* that is, How many pounds, ſhillings, or pence, would have been gained in the ſale upon 100 l. 100 s. or 100 d. laid out in the purchaſe? *Anſ.* 25 *per cent.* For,

L. L.
As 60 : 75 :: 100 : 125. Or, As 60 : 15 :: 100 : 25

Queſt. 5. If a halfpenny in the ſhilling be a merchant's profit, what does he gain *per cent.?* *Anſ.* 4 l. 3 s. 4 d. per 100 l. or $4\frac{1}{6}$ *per cent.*

Becauſe $\frac{1}{2}$ d. per ſhilling is 10 d. per l. you may work thus.

```
L. 100
     10
12)1000 pence.
  2|0)8|3  4
   L. 4  3  4
```

Or, Becauſe $\frac{1}{2}$ d. is $\frac{1}{24}$ s. and $24 = 4 \times 6$, you may work alſo thus.

```
4)100 l.
 6)25
  L. 4  3  4 per 100 l.
```

Hence a penny in the ſhilling is L. 8 : 6 : 8 per 100 l. or $8\frac{1}{3}$ *per cent.* And a penny halfpenny in the ſhilling is 12 l. 10 s. per 100 l. or $12\frac{1}{2}$ *per cent.*

Queſt. 6. If 2 d. in the ſhilling be a dealer's profit, what does he gain *per cent.?* *Anſ.* L. 16 : 13 : 4 per 100 l. or $16\frac{2}{3}$ *per cent.*

Becauſe 2 d. is $\frac{1}{6}$ s. take $\frac{1}{6}$ of 100 l. as follows.

```
6)100 l.
L. 16  13  4
```

Hence it is obvious, that whatever part of a ſhilling the profit or loſs is, the ſame part of 100 will be the gain

gain or loſs *per cent*. Thus, 3 d. of profit or loſs on the ſhilling is 25 *per cent*. And in like manner, from the gain or loſs *per cent*. given, may the profit or loſs per ſhilling be found.

And whatever part of a pound the given gain or loſs is, the like part of 100 will be the gain or loſs *per cent*.; which, therefore, may be found as directed above.

Queſt. 7. A chapman has goods to the value of L. 415 : 12 : 6; but coming to a bad market, was obliged to ſell them at 12 *per cent*. loſs: What were they ſold for? *Anſ*. 365 l. 15 s. See the work.

	L.	s.	d.
100			
12			
88			
If 100 : 88 ::	415	12	6
		20	
	8312		
	12		
	99750		
	88		
	79800		
	79800		
	12)87780.00		
	2|0)731|5		
Anſ. L.	365	15	

Or thus:

	L.	s.	d.
If 100 : 12 ::	415	12	6
			12
	L. 49.87	10	
	20		
	s. 17.50		
	12		
	d. 6.00		

	L.	s.	d.
Prime coſt	415	12	6
Loſs	49	17	6
Price	L. 365	15	0

Queſt. 8. A merchant buys goods to the value of 425 l. and offers to ſell them for 15 *per cent*. profit: What come they to? *Anſ*. 488 l. 15 s. Say,

L. If 100 : 115 :: 425 115 2125 425 425 L. 488|75 20 s. 15|00 L. s. *Anf.* 488 15	Or thus: 425 at 15 *per cent.* 4 400 | 60 25 | 3 15 63 15 profit. 455 prime coft. L. 488 15 price.

Queft. 9. The prime coft of a parcel of goods fent to Jamaica is L. 37 : 8 : $1\frac{1}{2}$; and being fold at 75 *per cent.* advance on the invoice, what does that amount to? *Anf.* L. 65 : 9 : $2\frac{1}{2}$. Say,

L. s. d. If 100 : 75 :: 37 8 $1\frac{1}{2}$ 20 748 12 8977 4 35910 75 17955 25137 4)26932.50 12)6733 2|0)56|1 1 Adv. L. 28 1 1 Prime coft 37 8 $1\frac{1}{2}$ Amount 65 9 $2\frac{1}{2}$	L. s. d. Or, If 100 : 175 :: 37 8 $1\frac{1}{2}$ 20 748 12 8977 4 35910 175 17955 25137 3591 4)62842.50 12)15710 2 2|0)130|9 2 Amount L. 65 9 $2\frac{1}{2}$

The 50 cut off is $\frac{50}{100}$ f. = $\frac{1}{2}$ f. more.

Queft.

Quest. 10. If 86 pieces of cloth cost in all 129 l. how must they be sold per piece to gain 15 *per cent.*? *Ans.* L. 1 : 14 : 6. Say,

		L.		L.	s.
If 100 :	115 : :	129 :		148	7.

	Pc.	L.	s.	Pc.	L.	s.	d.
And, If	86 :	148	7 : :	1 :	1	14	6

Quest. 11. A grocer bought 4 C. 1 Q. pepper for L. 15 : 17 : 4; which happening to be damnified, he is willing to lose 12½ *per cent.*: How must he sell it per lb.? *Ans.* At 7 d. per lb.

From 100 l. subtract 12 l. 10 s. and there remains 87 l. 10 s. Then say,

L.	L.	s.	L.	s.	d.	L.	s.	d.	
If 100 :	87	10 : :	15	17	4 :	13	17	8	the selling price of the whole.

	C.	Q.	L.	s.	d.	lb.	d.	
And, If	4	1 :	13	17	8 : :	1 :	7	the rate per lb.

Quest. 12. If a merchant, in selling goods at 8 d. per lb. gain 12 *per cent.* what will he gain *per cent.* in selling the same goods at 9 d. per lb.? *Ans.* 26 *per cent.*

d.		d.	
If 8 :	112 : :	9 :	126

Quest. 13. Sold a puncheon of rum for L. 30 l. and gained 20 *per cent.*: What was the prime cost? *Ans.* 25 l.

		L.	L.
If 120 :	100 : :	30 :	25

Quest. 14. A plummer sold 10 fodder of lead, each fodder containing 19½ C. for 204 l. 15 s. and gained 12½ *per cent.*: How much did it cost him in all? and how much per C.? *Ans.* 182 l. in all; and 18 s. 8 d. per C.

Queſt. 15. A merchant ſells 8 tuns of wines for 440 l. and loſes 12 *per cent.*: How much did it coſt per tun? and how does he ſell it per gallon? *Anſ.* It coſt 62 l. 10 s. per tun; and he ſells it at 4 s. 4 d. $1\frac{1}{2}\frac{1}{1}$ f. per gallon.

Queſt. 16. If a merchant ſell cloth at 11 s. 6 d. per yard, and thereby gain 15 *per cent.* what would he gain *per cent.* by ſelling the ſame cloth at 12 s. per yard? *Anſ.* 20 *per cent.*

Queſt. 17. A merchant ſelling corn at 8 s. per buſhel, gained 10 *per cent.*; but the market falling, he was obliged to ſell the reſt of the ſame corn at 7 s. per buſhel: What did he gain or loſe *per cent.* on this laſt ſale? *Anſ.* He loſt $3\frac{3}{4}$ *per cent.*

Queſt. 18. A draper bought 28 pieces of ſtuffs, at 4 l. per piece, and ſells 10 of them at 4 l. 6 s. per piece, and 8 of them at 4 l. 8 s. per piece: At what rate muſt he ſell the reſt, to gain 10 *per cent.* on the whole? *Anſ.* At 4 l. 10 s. per piece.

CHAP. VI.

MISCELLANIES.

I. *Brokage.*

BRokers are agents employed by merchants in tranſacting mercantile affairs; ſuch as, buying, ſelling, negotiating bills, &c. They are generally men that have been bred merchants, underſtand trade and exchange, with the proper ſeaſons of doing buſineſs. They muſt too be men of character; and no decayed merchant is allowed to officiate as broker without a ſpecial licence. The payment or fee for their ſervice is called *brokage*, being a ſcanty allowance of a few ſhillings *per cent.* or on the 100 l.; and is readily caſt up by practice, as in the following examples.

RULE.

Find the brokage at 1 *per cent.* by dividing the given ſum by 100, and then take aliquot parts of the quot.

1. What is the brokage of L. 985 : 15 : 6, at 6 s. *per cent.* ?

L.	9	85 15 6
		20
s.	17	15
		12
d.	1	86
		4
f.	3	44

	L.	*s.*	*d.*
6 s. $= \frac{3}{10}$ l.	9	17	$1\frac{3}{4}$
			3
10)	29	11	$5\frac{1}{4}$
Ans.	2	19	$1\frac{1}{2}$

Or,

	L.	*s.*	*d.*
	9	17	$1\frac{3}{4}$
5 s. $= \frac{1}{4}$	2	9	$3\frac{1}{4}$
1 s. $= \frac{1}{5}$		9	$10\frac{1}{4}$
	2	19	$1\frac{1}{2}$ *Ans.*

2. What is the brokage of 439 l. 19 s. at 7 s. 6 d. *per cent.* ?

L.	4	39 19
		20
s.	7	99
		12
d.	11	88
		4
f.	3	52

7 s. 6 d. $= \frac{3}{8}$ l.

	L.	s.	d.
	4	7	$11\frac{3}{4}$
			3
8)	13	3	$11\frac{1}{4}$
	1	12	$11\frac{3}{4}$ *Ans.*

3. What is the brokage of 345 l. 16 s. at 4 s. 9 d. *per cent.* ?

L.

L. 3.	45 16
	20
s. 9	16
	12
d. 1	92
	4
f. 3	68

s. d.	L.	s.	d.
	3	9	$1\frac{3}{4}$
$4\ 0 = \frac{1}{5}$		13	$9\frac{3}{4}$
$6 = \frac{1}{8}$		1	$8\frac{1}{2}$
$3 = \frac{1}{2}$			$10\frac{1}{4}$
		16	$4\frac{1}{2}$ *Anf.*

MORE EXAMPLES.

4. What is the brokage of L. 834 : 18 : 4, at 5s. 6d. *per cent.?* *Anf.* L. 2 : 5 : $10\frac{3}{4}$.

5. What is the brokage of L. 476 : 12 : 9, at 6s. 8d. *per cent.?* *Anf.* L. 1 : 11 : $9\frac{1}{4}$.

6. What is the brokage of L. 954, 17 s. at 4 s. 9 d. *per cent.?* *Anf.* L. 2 : 5 : 4.

II. *Bankruptcy.*

When a perfon in trade fails, ftops payment, or turns infolvent, he is called *bankrupt*; and the computations ufed in fettling affairs among the creditors are fuch as occur in the following queftions.

1. A bankrupt's effects amount to 811 l. 10 s. and he owes

	L.	s.	d.
To A	220	16	6
To B	312	0	0
To C	117	12	6
To D	106	12	6
To E	200	6	0
To F	124	12	6
In all	1082	0	0

How much can he afford to pay per pound? and what muft each creditor have?

Anf. 15 s. per pound; and the creditors get as follows.

L.

	L.	s.	d.
A	165	12	$4\frac{1}{2}$
B	234	0	0
C	88	4	$4\frac{1}{2}$
D	79	19	$4\frac{1}{2}$
E	150	4	6
F	93	9	$4\frac{1}{2}$
Proof	811	10	

Firſt find the dividend for 1 l. by ſaying,

	L.		L.	s.		L.
If	1082	:	811	10	::	1
				20		

```
1082)16230(15 s.
     1082
     ----
      5410
      5410
      ----
```

Then find the dividend to each creditor by practice, as follows.

s.		L.	s.	d.	
	A	220	16	6,	at 15 s.
$10 = \frac{1}{2}$		110	8	3	
$5 = \frac{1}{2}$		55	4	$1\frac{1}{2}$	
		165	12	$4\frac{1}{2}$	

s.		L.	
	B	312,	at 15 s.
$10 = \frac{1}{2}$		156	
$5 = \frac{1}{2}$		78	
		234	

Work in like manner for each of the other creditors.

Or you may work by decimal practice, viz. turn the dividend for 1 l. into a decimal, multiply it through the nine digits; and you have a table conſtructed, from which may be collected the dividend for each creditor, by moving the decimal point, and taking aliquot parts, as follows.

L.

	L.	s.	d.		L.
A	220	16	6		
	200	00	0	=	150.
	20	00	0	=	15.
		10	0	=	.375
		5	0	=	.1875
		1	0	=	.0375
			6	=	.01875
					165.61875

TABLE.

1	.75
2	1.50
3	2.25
4	3.00
5	3.75
6	4.50
7	5.25
8	6.00
9	6.75

L.	
312	
300 =	225.
10 =	7.5
2 =	1.5
	234

Or, Multiply 312 by .75

$$\begin{array}{r} 312 \\ .75 \\ \hline 1560 \\ 2184 \\ \hline 234.00 \end{array}$$

2. A breaks, and agrees with his creditors at 17 s. 6 d. per pound: What will B receive, to whom he owes 400 l.

s.	d.		L.
			400, at 17 s. 6 d.
10	0	=	200
5	0	=	100
2	6	=	50
			350 *Anʃ.*

3. A compounds with his creditors for 13 s. 4 d. per pound: What may B expect, to whom he owes 415 l. 9 s. 6 d.?

L.

	L.	*s.*	*d.*	
	415	9	6,	at 13 s. 4 d.
s. *d.*				
6 8 = $\frac{1}{3}$ =	138	9	10	
6 8 = $\frac{1}{3}$ =	138	9	10	
	276	19	8	*Anſ.*

MORE EXAMPLES.

4. A agrees with his creditors at 5 s. 5 d. per pound: What will B get, to whom he was owing 1000 l.? *Anſ.* L. 270 : 16 : 8.

5. A bankrupt's effects afford 11 s. 7½. per pound: What will B get, to whom there is owing 300 l.? *Anſ.* L. 174 : 7 : 6.

6. A bankrupt ſettles with his creditors at 6 s. 8 d. per pound, and clears with B, by paying him 317 l. 6 s. 8 d.: What was the original debt? *Anſ.* 952 l.

III. *Inſurance.*

Inſurance is a ſecurity given by the inſurers or underwriters, to indemnify the inſured from ſuch loſſes as are mentioned in the policy of inſurance, in conſideration of a ſum of money called *premium*; which varies according to the riſk, and is generally ſo much *per cent.*

It is an uſual article in the policy, that in caſe of loſs, the inſurer is to be allowed a ſmall diſcount or abatement, commonly 2 *per cent.*; that is, he is to pay but 98 l. for every 100 l. inſured; and the 98 l. is called the *ſhort recovery.*

A merchant ſometimes inſures, not only the full value of his cargo, but even the premium and diſcount; that, in caſe of loſs, he may be intitled to a ſum from the underwriters, or inſurance office, exactly equal to the value of the ſaid cargo; and this is called *covering his outſet.*

In caſe of damage or loſs, an eſtimate thereof is made out, and the underwriters pay in proportion to their ſeveral ſubſcriptions; and this is called *an average.*

Moſt of the computations relative to inſurance, fall under one or other of the caſes following.

Caſe I. Given the rate *per cent.* to find the premium.
Work by practice or aliquot parts.

Ex. What premium muſt be paid for inſuring 854 l. at $7\frac{1}{2}$ *per cent.?* and for inſuring 2860 l. at $13\frac{1}{2}$ *per cent.?*

Ex. 1.

p. c.		854	
5	$= \frac{1}{20}$	42.7	
$2\frac{1}{2}$	$= \frac{1}{2}$	21.35	L. s.
$7\frac{1}{2}$		64.05	=64 1 *Anſ.*

Ex. 2.

p. c.		2860	
10	$= \frac{1}{10}$	286	
2	$= \frac{1}{5}$	57.2	
1	$= \frac{1}{2}$	28.6	
$\frac{1}{2}$	$= \frac{1}{2}$	14.3	L. s.
$13\frac{1}{2}$		386.1	=386 2 *Anſ.*

Caſe II. Given the diſcount *per cent.* to find the ſhort recovery.

Work by aliquot parts.

Ex. What is the ſhort recovery of L. 2000 inſured, when diſcount is allowed at 2 *per cent.?* and at $2\frac{1}{2}$ *per cent.?*

p. c.			
2	$= \frac{1}{50}$	2000	
		40	diſcount.
		1960	ſhort recovery.

p. c.			
$2\frac{1}{2}$	$= \frac{1}{40}$	2000	
		50	diſcount.
		1950	ſhort recovery.

Caſe III. Given the rate *per cent.* of premium and diſcount, to find what ſum muſt be inſured to cover the outſet in a ſingle voyage.

Subtract the ſum of the premium, and diſcount from 100, and then ſay, As the remainder to 100, ſo the outſet to the anſwer.

Ex. 1. If the premium be 10 *per cent.* and the diſcount 2 *per cent.* what ſum muſt be inſured to cover L. 100 outſet?

10 premium.	100				
2 difcount.	12				
—	——	L.	L.	s.	d.
12 fum	As 88 : 100 ::	100 :	113	12	8 *Anf.*

Proof.

	L.	s.	d.	
Sum infured	113	12	8	
Deduct 2 *per cent.*	2	5	5	difcount.
Infurer pays	111	7	3	in cafe of lofs.
Deduct 10 *per cent.*	11	7	3	premium.
Outfet covered	100			or made good.

Ex. 2. If the premium be $13\frac{1}{2}$ *per cent.* and the difcount 2 *per cent.* what fum muft be infured in order to cover 800 l. outfet?

$13\frac{1}{2}$ prem.	100				
2 difc.	15.5				
——	——	L.	L.	s.	d.
$15\frac{1}{2}$ fum.	As 84.5 : 100 ::	800 :	946	14	$10\frac{3}{4}$ *Anf.*

Cafe IV. Given the rate *per cent.* of premium and difcount, to find the premium out and home, and what fum muft be infured to cover the infet.

Firft, Find the premium on the voyage out, as taught above, and for the voyage home work thus: From the fhort recovery deduct the premium; then fay, As the remainder to the premium, fo the fum of the outfet and premium of the voyage out, to the premium of the voyage home; which, added to the premium of the voyage out, gives the total premium. Laftly, Find what fum infured will cover the infet by cafe 3.

Ex. If the premium be 10 *per cent.* and the difcount 2 *per cent.* what premium muft be paid for infuring 100 l. out and home? and what fum muft be infured to cover the infet?

By caſe 3. ex. 1. the premium of the voyage out is L. 11 : 7 : 3.

		L	s.	d.				
	98	11	7	3	premium out.			
	10	100	0	0	outſet.			
					L.	s.	d.	
As	88 : 10 ::	111	7	3 :	12	13	1	prem. home.
					11	7	3	prem. out.
					24	00	4	total prem.

Laſtly, by caſe 3. find what ſum muſt be inſured to cover L. 111 : 7 : 3, viz. ſay,

	L.	s.	d.				
As 88 : 100 ::	111	7	3				
			10				
	1113	12	6				
			10	L.	s.	d.	
88)	11136	5	0(	126	10	11	*Anſ.*

In former times the policies of inſurance were generally conceived in ſuch vague terms, that ſometimes it was no eaſy matter to ſettle an average when a loſs happened. The affair was uſually brought before a court, critical queſtions were ſtarted, difficulties and clouds of duſt were raiſed, that tended to inveigle the cauſe, and render it a ſubject of endleſs altercation : but the caſe is otherwiſe now; for ſcarce any misfortune can happen but what is expreſsly ſpecified in the policies uſed at preſent. I ſhall conclude inſurance with one example of a general average on ſhip, goods, and freight.

A ſhip, by ſtreſs of weather, loſt her maſts, was forced aſhore, and the damage eſtimated at 36 l.

A's

	L.
A's goods, - -	1200
B's goods, - -	480
Insured by Mess. Adair,	1680
A's own risk, - -	60
B's own risk, - -	40
Value of the goods,	1780
Value of the ship, -	600
Amount of freight,	200
	2580

Now, if 2580 l. pays 36 l. then,

L.		L.	s.	d.
1680	pays -	23	8	10
60	pays -	0	16	9
40	pays -	0	11	2
600	the ship, pays	8	7	$5\frac{1}{4}$
200	freight, pays	2	15	$9\frac{3}{4}$
	Proof	36	0	0

IV. *Of purchasing stocks.*

The British government, ever since the revolution in 1688, have been in use to raise supplies, for carrying on war, or answering other exigencies of the state, by borrowing money, and appropriating the public taxes for payment of the interest.

These national debts, contracted by borrowing from the bank of England, from trading companies, and from private persons, or by selling annuities, with the provisions made by parliament for paying the interest due thereon, or for cancelling the debts, are commonly called the *public funds* or *stocks*; and come under various designations; such as, *Bank Stock*, *India Stock*, *India Annuities*, *India Bonds*, *South-Sea Stock*, *South-Sea Annuities*, *Three per cent. Reduced Annuities*, *Three per cent. Consolidated Annuities*, *Three and a half*

half per cent. Annuities, Four per cent. Annuities, Long Annuities, Exchequer Bills, Navy and *Victualling Bills*, &c.

When the parliament has voted the ſupplies, a ſubſcription is opened, and generally a few men of fortune ſubſcribe for the whole ; and as the ſubſcriptions are all transferable, they afterwards ſell off theſe ſtocks in ſmall ſhares, and at high rates, and the profits thence ariſing are ſo much clear gain.

The annuities and growing intereſt have hitherto been always punctually paid ; and as the ſtocks now amount to many millions Sterling, not only monied corporations and men of fortune in Britain, but many foreigners have come to be concerned in them. The Britiſh ſtocks are now become a conſiderable branch of commerce, carried on in Exchange alley ; and commiſſions are ſent up from all the parts of Europe to make purchaſes in them.

Stocks are bought or ſold at ſo much *per cent.* ; and the price, or ſum to be paid, may be caſt up by practice, or decimally, as in the following examples.

1. What muſt be paid for 403 l. 18 s. bank annuities, at 94 *per cent.* ?

By practice.

	L.	s.		
	403	18,	at 94 *per cent.*	
$20 = \frac{1}{5}$	80	15	7	
$5 = \frac{1}{4}$	20	3	$10\frac{3}{4}$	
$1 = \frac{1}{5}$	4	0	$9\frac{1}{4}$	
6	24	4	8	ſub.
	379	13	4	*Anſ.*

Decimally.

Decimally.

$$\begin{array}{r} \text{L.} \\ \text{As } 100 : 94 :: 403.9 \\ 94 \\ \hline 16156 \\ 36351 \\ \hline \textit{Anf.}\ 379.666 \end{array}$$

2. What fum will purchafe 875 l. 10 s. bank ftock, at $131\frac{1}{4}$ *per cent.*?

By practice.

		L.	s.		
100 =		875	10		at $131\frac{1}{4}$ *per cent.*
25 =	$\frac{1}{4}$	218	17	6	
5 =	$\frac{1}{5}$	43	15	6	
$1\frac{1}{4}$ =	$\frac{1}{4}$	10	18	$10\frac{1}{2}$	
$131\frac{1}{4}$		1149	1	$10\frac{1}{2}$	*Anf.*

Decimally.

$$\begin{array}{r} \text{L.} \\ \text{As } 100 : 131.25 :: 875.5 \\ 875.5 \\ \hline 65625 \\ 65625 \\ 91875 \\ 105000 \\ \hline 1149.09375 \quad \textit{Anf.} \end{array}$$

3. What fum will purchafe 432 l. 15 s. South-Sea ftock, at $113\frac{5}{8}$ *per cent.*?

By

By practice.

		L.	s.	d.
100 =		432	15	0
10 =	$\frac{1}{10}$	43	5	6
2 =	$\frac{1}{5}$	8	13	1
1 =	$\frac{1}{2}$	4	6	$6\frac{1}{2}$
$\frac{1}{2}$ =	$\frac{1}{2}$	2	3	$3\frac{1}{4}$
$\frac{1}{8}$ =	$\frac{1}{4}$	0	10	$9\frac{3}{4}$
$113\frac{5}{8}$		491	14	$2\frac{1}{2}$ *Anſ.*

Decimally. L.

As 100 : 113.625 :: 432.75
Mult. 57.234 inverted.

```
454500
 34087
  2272
   795
    56
------
491.710
```

If the ſtock be any juſt number of hundreds, multiply the rate *per cent.* by the number of hundreds, and the product is the price, as in the following example.

4. What muſt I pay for 800 3 *per cent.* annuities, at $124\frac{7}{8}$ *per cent.*?

L.	s.	d.
124	17	6
		8
999	0	0 *Anſ.*

V. *An eaſy way of finding the value of the ſhort hundred, or five ſcore.*

RULE I.

If the rate, or price of 1, be ſhillings, multiply the rate by 5, and the product is the anſwer in pounds.

The

The anſwer, multiplied by 2, 3, 4, &c. or by 10, gives the price of 200, 300, 400, &c. or of 1000.

EXAMPLES.

1. What coſt 100 ells, at 16 s. per ell?

$$\begin{array}{rl} & 16\text{ s.} \\ & \underline{5} \\ \text{L.} & 80 \; \textit{Anſ.} \\ & \underline{3} \\ & 240 \text{ price of 300 ells.} \end{array}$$

2. What coſt 100 yards, at 35 s. per yard?

$$\begin{array}{rl} & 35\text{ s.} \\ & \underline{5} \\ \text{L.} & 175 \; \textit{Anſ.} \\ & \underline{10} \\ & 1750 \text{ price of 1000 yds.} \end{array}$$

3. What coſt 100 lb. at 3 l. 14 s. per lb.?

$$\begin{array}{rll} & \text{L.} & \text{s.} \\ & 3 & 14 \\ & \underline{20} & \\ & 74 & \\ & \underline{5} & \\ \text{L.} & 370 & \textit{Anſ.} \end{array}$$

4. What coſt 100 gallons, at 2 l. 17 s. per gallon?

$$\begin{array}{rll} & \text{L.} & \text{s.} \\ & 2 & 17 \\ & \underline{20} & \\ & 57 & \\ & \underline{5} & \\ \text{L.} & 285 & \textit{Anſ.} \end{array}$$

RULE II.

If the rate be pence, multiply the rate by 5, divide the product by 12, the quot will be pounds, and every unit of the remainder 1 s. 8 d.

EXAMPLES.

1. What will 100 yards coſt, at 9 d. per yard?

$$\begin{array}{l} \quad 9\text{ d.} \\ \quad \underline{5}\ \text{L.}\ \ \text{s.} \\ 12)45(3\ \ 15\ \textit{Anſ.} \\ \quad \underline{36} \\ \quad (9) \end{array}$$

2. What will 100 ells coſt, at 11 d. per ell?

$$\begin{array}{l} \quad 11\text{ d.} \\ \quad \underline{5}\ \text{L.}\ \ \text{s.}\ \ \text{d.} \\ 12)55(4\ \ 11\ \ 8\ \textit{Anſ.} \\ \quad \underline{48} \\ \quad (7) \end{array}$$

RULE III.

If the rate be ſhillings and pence, multiply the rate by 5; the product under ſhillings will be pounds, and every unit of the product under pence will be 1 s. 8 d.

EXAMPLES.

1. What will 100 ells coſt, at 4 s. 5 d. per ell?

	s.	d.
	4	5
		5
	22	1
Anſ. L. 22	1	8

2. What will 100 yard coſt, at 7 s. 6 d. per yard?

	s.	d.
	7	6
		5
	37	6
Anſ. L. 37	10	

RULE IV.

If the rate be farthings, multiply the rate by 5, and every unit of the product will be 5 d.

EXAMPLES.

1. What will 100 apples coſt, at 1 f. each?

1 f.
5 d. s. d.
Anſ. 5 × 5 = 25 = 2 1

2. What will 100 oranges coſt, at 3 f. each?

3 f.
5 d. s. d.
15 × 5 = 75 = 6 3 *Anſ.*

If the value of 100 be given, and the rate, or price of 1, be required, reverſe the operation.

VI. *An eaſy way of finding the value of the long hundred, or ſix ſcore.*

RULE.

Eſteem the pence in the rate ſo many pounds; and for the farthings, take ſuch a part of a pound as they are

are of a penny, and the half of this amount is the answer.

EXAMPLES.

1. What will 120 ells cost, at $7\frac{1}{2}$ d. per ell?

	L.	s.	
2)	7	10	
L.	3	15	*Anf.*

2. What will 120 yards cost, at $13\frac{1}{4}$ d. per yard?

	L.	s.		
2)	13	5		
L.	6	12	6	*Anf.*

3. What will 120 gallons cost, at $16\frac{3}{4}$ d. per gallon?

	L.	s.		
2)	16	15		
L.	8	7	6	*Anf.*

4. What will 120 lb. cost, at 17 s. $3\frac{1}{2}$ per lb.?

		12	
2)	207	10	
L.	103	15	*Anf.*

If the price of 120 be given, and the rate be required, reverse the operation.

VII. *A short way of finding the value of* 1 *C. or* 112 *lb.*

RULE.

Esteem every penny in the rate 9 s. 4 d; esteem three farthings 7 s.; esteem a halfpenny 4 s. 8 d.; and a farthing 2 s. 4 d.

EXAMPLES.

1. What will 1 C. cost, at $7\frac{1}{2}$ d. per lb.?

	L.	s.	d.	
1 d. =	0	9	4	
			7	
7 d. =	3	5	4	
$\frac{1}{2}$ d. =	0	4	8	
L.	3	10	0	*Anf.*

2. What will 1 C. cost, at $9\frac{3}{4}$ d. per lb.?

	L.	s.	d.	
1 d. =	0	9	4	
			9	
9 d. =	4	4	0	
$\frac{3}{4}$ d. =	0	7	0	
L.	4	11	0	*Anf.*

VIII. *A ſhort way of finding the value of the great groſs.*

RULE.

Multiply the rate or price of 1 dozen in pence, by 3, divide the product by 5, and the quot will be the anſwer in pounds.

EXAMPLES.

1. What will one great groſs coſt, at 7 d. per dozen?

$$\begin{array}{r} 7\text{ d.} \\ 3 \\ \hline 5)21 \\ \hline \text{L. } 4\ \ 4 \end{array}$$ *Anſ.*

2. What will 1 great groſs coſt, at 3 s. 6 d. per doz.?

$$\begin{array}{r} 42\text{ d.} \\ 3 \\ \hline 5)126 \\ \hline \text{L. } 25\ \ 4 \end{array}$$ *Anſ.*

In the ſame manner may the value of the ſmall groſs be found from the price of 1 given.

If the value of the groſs be given, and the rate be required, reverſe the operation; that is, multiply by 5, and divide the product by 3, and the quot is the anſwer in pence.

IX. *The price of goods caſt up by inverting the queſtion.*

In many caſes, the value or price of goods may be found with great eaſe, by inverting the queſtion, viz. eſteem the given quantity to be the rate, and the rate to be the quantity.

EX-

EXAMPLES.

1. What coſt 15 yards, at 7 d. per yard?

Invert the queſtion, by ſaying, What coſt 7 yards, at 15 d. or 1 s. 3 d.; and then work as follows.

	s.	*d.*	
	1	3	
		7	
s.	8	9	*Anſ.*

2. What coſt 53 ells at 9 d. per ell? that is, 9 ells at 53 d.?

		s.	*d.*	
		4	5	
			9	
L.	1	19	9	*Anſ.*

3. What coſt $58\frac{1}{2}$ gallons, at 11 d. per gallon? or 11 gallons, at $58\frac{1}{2}$ d.?

		s.	*d.*	
		4	$10\frac{1}{2}$	
			11	
L.	2	13	$7\frac{1}{2}$	*Anſ.*

4. What coſt $42\frac{1}{4}$ hogſheads, at 28 s. per hogſhead? or 28 hogſheads at $42\frac{1}{4}$ s.?

		L.	*s.*	*d.*	
		2	2	3	
$28 = 7 \times 4$				7	
		14	15	9	
				4	
	L.	59	3	0	*Anſ.*

CHAP. VII.

EXCHANGE.

EXchange is the receiving or paying money in one country for the like ſum in another.

The par in exchange is when the ſum received and paid away are of the ſame intrinſic value, or contain an equal quantity of pure gold or ſilver.

The courſe of exchange is the current price betwixt two places, which is always fluctuating and unſettled, being ſometimes above and ſometimes below par, according to the circumſtances of trade.

When the courſe of exchange riſes above par, the country where it riſes may conclude for certain, that the

the balance of trade runs againſt them. The truth of this will appear, if we ſuppoſe Britain to import from any foreign place goods to the value of 100,000 l. at par, and export only to the value of 80,000 l.; in this caſe bills on the ſaid foreign place will be ſcarce in Britain, and conſequently will riſe in value; and after the 80,000 l. is paid, bills muſt be procured from other places at a high rate to pay the remainder, ſo that perhaps 120,000 l. may be paid for bills to diſcharge a debt of 100,000 l.

Though the courſe of exchange be in a perpetual flux, and riſes or falls according to the circumſtances of trade, yet the exchanges of London, Holland, Hamburgh, and Venice, in a great meaſure regulate thoſe of all other places in Europe.

I. *Exchange with Holland.*

MONEY-TABLE.

			Par in Sterling.		*s.*	*d.*
8	Pennings, or 2 duytes,	make	1 groat or penny	=	0	0.54
2	Groats, or 16 pennings,	make	1 ſtiver	=	0	1.09
6	Stivers, or 12 pence,	make	1 ſchilling	=	0	9.56
20	Schillings,	make	1 pound Flemiſh	=	10	11.18
20	Stivers, or 40 pence,	make	1 guilder or florin	=	1	9.86
6	Guilders, or florins,	make	1 pound Flemiſh	=	10	11.18
$2\frac{1}{2}$	Guilders, or florins,	make	1 rixdollar	=	4	6.66

In Holland there are two ſorts of money, bank and current. The bank is reckoned good ſecurity; demands on the bank are readily anſwered; and hence bank money is generally rated from 3 to 6 *per cent.* better than the current. The difference between the bank and current money is called the *agio*.

Bills on Holland are always drawn in bank money; and if accounts be ſent over from Holland to Britain in current money, the Britiſh merchant pays theſe accounts by bills, and in this caſe has the benefit of the agio.

PROB.

PROB. I.

To reduce bank money to current money.

RULE.

As 100 to 100 + agio, so the given guilders to the answer.

EXAMPLE.

What will 2210 guilders in bank money amount to in Holland currency, the agio being $3\frac{1}{8}$ *per cent.*?

```
                          Guild.
As 100  :  103⅛  ::  2210
     8        3        825
   ---      ---      -----
   800      825      11050
                      4420
                     17680
                    --------- Guild. st. pen.
            8|00)18232|50(2279  1    4 cur.
                 16···  20
                 ---   ----
                  22   10|00(
                  16    8
                  --   --
                   63   2
                   56  16
                   --  --
                   72  32
                   72  32
                   --  --
```

Or, by practice,

```
50)2210
      44.2     = 2 per cent.
      22.1     = 1 per cent.
       2.7625  = ⅛ per cent.
   ----------
   2279.0625
```

If the agio only be required, make the agio the middle term, thus:

Guil.

Guil. *Guil.* *ſt.* *pen.*

As 100 : $3\frac{1}{8}$ :: 2210 : 69 1 4 agio. Or, work by practice, as above.

PROB. II.

To reduce current money to bank money.

RULE.

As 100 + agio to 100, ſo the given guilders to the anſwer.

EXAMPLE.

What will 2279 guilders 1 ſtiver 4 pennings, Holland currency, amount to in bank money, the agio being $3\frac{1}{8}$ *per cent.*?

```
  Guild.   Guild.   Guild. ſt. pen.
As 103⅛  :  100  ::  2279  1  4
      8       8          20
   ----    ----      -------
    825     800       45581
     20                  16
   -----             -------
  16500              273490
    16                45581
   -----             -------
    990              729300
   165                  800
  -------            ---------
8)264|000          8)583440|000
  3)33               3)72930 Guild.
    11              11)24310(2210 bank
```

In Amſterdam, Rotterdam, Middleburgh, &c. books and accounts are kept by ſome in guilders, ſtivers, and pennings, and by others in pounds, ſchillings, and pence, Flemiſh.

Britain gives 1 l. Sterling for an uncertain number of ſchillings and pence Flemiſh. The par is 1 l. Sterling for 36.59 s. Flemiſh; that is, 1 l. 16 s. 7.08 d. Flemiſh.

When

When the Flemish rate rises above par, Britain gains and Holland loses by the exchange, and the contrary.

Sterling money is changed into Flemish, by saying,

As 1 l. Sterling to the given rate,

So is the given Sterling to the Flemish sought.

Or, the Flemish money may be cast up by practice.

Dutch money, whether pounds, schillings, pence Flemish, or guilders, stivers, pennings, may be changed into Sterling, by saying,

As the given rate to 1 l. Sterling,

So the given Dutch to the Sterling sought.

EXAMPLES.

1. A merchant in Britain draws on Amsterdam for 782 l. Sterling: How many pounds Flemish, and how many guilders will that amount to, exchange at 34 s. 8 d. per pound Sterling?

```
 L.    s. d.    L.
If 1 : 34  8 :: 782
           12
          ----
          416
          782
          ----
          832
        3328
       2912
       ------
12)325312
       ------ d.
2|0) 2710|9 4
       ------
L. 1355 9 4 Flem.
```

Decimally.

```
 L     s.        L.
If 1 : 34.6 :: 782
           782
          ----
           692
         27733
        242066
       --------
2|0)2710|9.2
       --------
L.1355 9 4 Flem.
```

By practice.

	L.	s.	d.
	782		
10 s. = $\frac{1}{2}$	391		
4 s. = $\frac{1}{5}$	156	8	
8 d. = $\frac{1}{6}$	26	1	4
	1355	9	4 Fl.

Or thus:

	L.	s.	d.
	782		
14 s. = $\frac{7}{10}$	547	8	
8 d. = $\frac{1}{30}$	26	1	4
	1355	9	4 Fl.

Multiply the Flemiſh pounds and ſhillings by 6, and the product will be guilders and ſtivers; and if there be any pence, multiply them by 8 for pennings; or, divide the Flemiſh pence by 40, and the quot will be guilders, and the half of the remainder, if there be any, will be ſtivers, and 1 penny odd will be half a ſtiver, or 8 pennings, as follows.

L.	s.	d.
1355	9	4
	6	
Guild. 8132	16 ſtiv.	

Flem. pence.
4|0)32531|2(32 rem.
Guild. 8132 16 ſtiv.

2. Change 591 l. 5 s. Flemiſh into Sterling money, exchange at 37 s. 6 d. Flemiſh per l. Sterling.

	Flem.		Ster.		Flem.	
	s.	d.	L.		L.	s.
If	37	6	: 1	::	591	5
	2					20
5)	75				11825	
5)	15					2
	3			5)	23650	
				5)	4730	
				3)	946	
					$315\frac{1}{3}$	

	L.	s.	d.
Anſ.	315	6	8 Ster.

Decimally.

Decimally.

5) L. L. 5) L.
If 1.875 : 1 : : 591.25

5) .375 5) 118.25
5) .075 5) 23.65
.015 .015) 4.73(315.3
45
23
15
80
75
50
45
* 5

3. Change 1693 guilders 6 ftivers into Sterling money, exchange at 34 s. 5 d. per pound Sterling.

Flem. Ster.
s. d. L. Guild. ftiv.
If 34 5 : 1 : : 1693 6
12 20

413 33866
2

413)67732(164 L. Ster.
413
2643
2478
1652
1652

Decimally.

```
34.41⅝
    .3
-------
10.3250 : 1 :: 1693.3
10.325)1693.300(164 L. Ster. Anſ.
        10325
        -----
         66080
         61950
         -----
          41300
          41300
          -----
```

In working decimally, becauſe 10 s. Flemiſh is equal to 3 guilders, we might ſay, As 10 : 3 :: 34.41⅝ to the guilders in the rate; but it is the ſame in effect, and ſhorter, to multiply by .3.

When the rate is 33 s. 4 d. Sterling, money is eaſily converted into guilders, or guilders into Sterling, the guilder in this caſe being equal to 2 s. Sterling.

4. A merchant in London remits to Rotterdam 1000 l. Sterling, exchange at 35 s. 4 d. and ſome time afterwards draws for 1000 l. Sterling, exchange at 34 s. 3 d. How much Flemiſh money has he ſtill in the bank, and how much Sterling money ought he to have drawn for to bring home the whole remittance?

```
s. d.                                      s.  d.
35 4     1000 = 10 × 10 × 10               1   1
34 3                                          10
----                                   ---------
 1 1                                      10  10
                                              10
                                       ---------
                                   L. 5   8   4
                                              10
                                   -------------
                                   54   3   4 Fl. in bank.
```

To

To find how much Sterling muſt be drawn for to bring home the remittance, ſay,

	s. d.		s. d.		L.St.		L. s. d.	
As	34 3	:	35 4	::	1000	:	1031 12 7	*St. Anſ.*
	12		12					
	411		424					

5. When the rate of exchange is 36 s. 8 d. what is the value of the guilder?

	s. d.		d.St.		d. Flem. in 1 guilder.
	36 8				
	12				
Fl. d.	4)440	:	240	::	40
	11		6		
			40		

$11)240(21\frac{9}{11}$ *d. Sterl. Anſ.*

6. When the value of the guilder is $21\frac{9}{11}$ d. Sterling, what is the rate of exchange?

	d. St.		d.Fl.		d. St.
If	$21\frac{9}{11}$	:	40	::	240
	11				11
	6)240				2640
	4) 4				44
	1				11
					40

s. d.
12)440)36 8 *Flem. Anſ.*

7. When the current guilder is worth 22 d. Sterling, what is the value of the bank guilder, the agio being 4 *per cent?*

If

d.
If 100 : 104 :: 22
22

208
208

100)2288(21.88 d. Sterl. *Anf.*

MORE EXAMPLES.

8. Britain draws on Holland for 1814 l. Sterling: How many guilders will that come to, exchange at 34 s. 8 d. Flemifh per l. Sterling? *Anf.* 18865 guild. 12 ftivers.

9. What may I draw for on London, if I pay in Amfterdam 3628 guilders, exchange at 35 s. $2\frac{1}{2}$ d. per l. Sterling? *Anf.* L. 343 : 9 : 7.

10. What Sterling money is equivalent to 687 guilders 5 ftivers, exchange at 33 s. 4 d. per l. Sterling? *Anf.* L. 68 : 14 : 6.

Holland exchanges with other nations as follows, viz. with

	Flem. d.		*Flem. d.*
Hamburgh, on the dollar,	= $66\frac{2}{3}$	Leghorn, on the piaftre,	= 100
France, on the crown,	= 54	Florence, on the crown,	= 120
Spain, on the ducat,	= $109\frac{4}{5}$	Naples, on the ducat,	= $74\frac{2}{3}$
Portugal, on the crufade,	= 50	Rome, on the crown,	= 136
Venice, on the ducat	= 93	Milan, on the ducat,	= 102
Genoa, on the pezzo,	= 100	Bologna, on the dollar,	= $94\frac{4}{9}$

Exchange betwixt Britain and Antwerp, as alfo the Auftrian Netherlands, is negotiated the fame way as with Holland, only the par is fomewhat different, as will be defcribed in article 2. following.

II.

II. *Exchange with Hamburgh.*

MONEY-TABLE.

			Par in Sterling.		s.	d.
12	Phennings	make	1	schilling-lub	= 0	$1\frac{1}{8}$
16	Schilling-lubs		1	mark	= 1	6
2	Marks		1	dollar	= 3	0
3	Marks		1	rixdollar	= 4	6
$6\frac{1}{4}$	Marks		1	ducat	= 9	$4\frac{1}{2}$

Books and accounts are kept at the bank, and by most people in the city, in marks, schilling-lubs, and phennings; but some keep them in pounds, schillings, and groots Flemish.

The agio at Hamburgh runs between 20 and 40 *per cent*. All bills are paid in bank-money.

Hamburgh exchanges with Britain by giving an uncertain number of schillings and groots Flemish for the pound Sterling. The groot or penny Flemish here, as also at Antwerp, is worth $\frac{54}{96}$ of a penny Sterling; and so something better than in Holland, where it is only $\frac{54}{100}$ d. Sterling.

				Flemish.
6	Phennings	make	1	groot or penny
6	Schilling-lubs		1	schilling
1	Schilling-lub		2	pence or groots
1	Mark		32	pence or groots
$7\frac{1}{2}$	Marks		1	pound

The par with Hamburgh, and also with Antwerp, is 35 s. $6\frac{2}{3}$ d. Flemish for 1 l. Sterling.

EXAMPLES.

1. How many marks must be received at Hamburgh for 300 l. Sterling, exchange at 35 s. 3 d. Flemish per l. Sterling?

L.

```
     L. s.  d.    L.
  If 1 : 35  3 :: 300
         12
       ----
        423
          300
      -------  M.  sch.
  32)126900(3965  10
       96...
       ----
       309
       288
       ----
        210
        192
        ----
         180
         160
        ----
        (20)
         16
        ----
       )320
        32
        ----
        (00)
```

Decimally.

```
          Flem. s. Marks. Flem. s.
          If 20  :  7.5  ::  35.25
                4  :  1.5  ::  35.25
                                1.5
                             ------
                              17625
                              3525
                             ------
                           4)52.875
                             --------
Marks in 1 l. Sterling       13.21875
                                  300
                             ---------
Marks in 300 l. Sterling     3965.62500
                                   16
                                 -----
                                 3750
                                 625
                                -----
        Schilling-lubs         10.000
```

2. How

2. How much Sterling money will a bill of 3965 marks 10 ſchilling-lubs amount to, exchange at 35 s. 3 d. Flemiſh per l. Sterling ?

```
   Fl. s.  d.  L.St.   Mks.  ſch.
If  35     3 : 1 :: 3965    10
    12                32     2
   ----             ----    --
   423              7930    20 d.
                   11897
                  ------
            423)126900(300 l. Sterling.
                 1269
                 ----
```

Decimally.

```
        4 : 1.5 :: 35.25
                     1.5
                  ------
                   17625
                   3525
                  ------
                4)52.875(13.21875
  13.21875)3965.62500(300 l. Sterling.
           3965625
           -------
```

3. Hamburgh is indebted to Britain 2648 marks current money : For how many marks may Britain draw on the bank, the agio being 30 *per cent.*

```
                          Marks.
As 130 : 100 :: 2648
    13 :  10      10
               ------  Mks. ſch. phen.
          13)26480(2036  14   9.  Anſ.
```

4. What is the Sterling value of a mark-lubs, when the exchange between Britain and Hamburgh is 36 s. 8 d. Flemiſh per l. Sterling ?

$$
\begin{array}{llll}
\textit{Flem. s.} & \textit{d.} & \textit{d. St.} & \textit{d. Flem.} \\
\text{If } 36 & 8 : 240 & :: & 32 \\
\underline{12} & & & \underline{6} \\
440 & & & 11)192(17\tfrac{5}{11} \text{ d. Ster. } \textit{Ans.} \\
44 & : 24 & & \underline{11} \\
11 & : 6 & & 82 \\
 & & & \underline{77} \\
 & & & (5)
\end{array}
$$

Becauſe 40 d. Flemiſh make a guilder of Holland, and 32 d. Flemiſh make a mark of Hamburgh, you may reduce mark to guilders, by ſaying, As 40 to 32, ſo the given marks to the guilders ſought. Or, from the given number of marks, ſubtract one fifth of itſelf, and the remainder is the anſwer.

EXAMPLE.

How many guilders of Holland are equivalent to 1730 marks of Hamburgh?

$$
\begin{array}{ll}
 & \textit{Marks.} \\
\text{If } 40 : 32 :: & 1730 \\
\quad 5 : 4 & \underline{\quad 4} \quad \textit{Guild.} \\
 & 5)6920(1384 \ \textit{Anſ.}
\end{array}
\qquad
\begin{array}{rl}
 & \text{Or thus:} \\
 & 1730 \text{ marks.} \\
\tfrac{1}{5} = & \underline{346} \\
 & 1384 \text{ guilders.}
\end{array}
$$

Hamburgh exchanges with other countries as follows, viz. with

Holland, on the dollar,	=	$66\frac{2}{3}$ d. Flemiſh.
France, on the crown,	=	$51\frac{4}{5}$ d. Flemiſh.
Spain, on the ducat,	=	$52\frac{4}{5}$ ſchilling-lubs.
Portugal, on the cruſade,	=	48 d. Flemiſh.
Venice, on the ducat,	=	$89\frac{3}{10}$ d. Flemiſh.
Leghorn, on the piaſtre,	=	96 d. Flemiſh.

III. *Exchange with France.*

MONEY-TABLE.

			Par in Ster.	s.	d.
12 deniers	make	1 ſol	=	0	$0\frac{39}{80}$
20 ſols	make	1 livre	=	0	$9\frac{3}{4}$
3 livres	make	1 crown	=	2	$5\frac{1}{4}$

At

At Paris, Rouen, Lyons, &c. books and accounts are kept in livres, fols, and deniers; and the exchange with Britain is on the crown, or ecu, of 3 livres, or 60 fols Tournois. Britain gives for the crown an uncertain number of pence, commonly between 30 and 34, the par, as mentioned above, being $29\frac{1}{4}$ d.

EXAMPLES.

1. What Sterling money muft be paid in London to receive in Paris 1978 crowns 25 fols, exchange at $31\frac{5}{8}$ d. per crown?

Sols.	*d.*		*Cr.*	*fols.*
If 60 :	$31\frac{5}{8}$	: :	1978	25
	253			60

```
             118705
                253
            -------
             356115
            593525
           237410
          ---------
     6|0)3003236|5  Rem.
        8)500539    3
       12)62567    11
        2|0)521|3  13
      L. 260  13  11 3/8  Ans.
```

By practice.

d.			*Cr.* 1978	*fols.* 25, at	$31\frac{5}{8}$ d.
30	=	$\frac{1}{8}$	247	5	0
$1\frac{1}{2}$	=	$\frac{1}{20}$	12	7	3
$\frac{1}{8}$	=	$\frac{1}{12}$	1	0	$7\frac{1}{4}$
Sols 20	=	$\frac{1}{3}$	0	0	$10\frac{1}{2}$
5	=	$\frac{1}{4}$	0	0	$2\frac{1}{2}$
			260	13	$11\frac{1}{4}$

If you work decimally, say,

	Cr.	*d. Ster.*		*Cr.*		*d. Ster.*
As	1	: 31.625	: :	1978.41$\dot{6}$	:	62567.42708$\dot{3}$

2. How many French livres will L. 121 : 18 : 6 Sterling amount to, exchange at $32\frac{7}{8}$ d. per crown ?

	d.		*Liv.*		*L.*	*s.*	*d.*
If	$32\frac{7}{8}$	:	3	: :	121	18	6
	263		8			20	
			24			2438	
						12	
						29262	
						24	
						117048	
						58524	

	Liv.	*sols.*	*den.*	
263)702288(	2670	5	11	*Ans.*

Rem. (78) = 5 sols 11 deniers.

5. What must I pay in Britain to receive in France 3965 livres, exchange at 31 d. Sterling per crown ?

Liv.	*d.*		*Liv.*		*L.*	*s.*	*d.*
3	: 31	: :	3956	:	170	6	$6\frac{2}{3}$

Or thus :

3)3956				
8)1318	13	4		
30)164	16	8	at 30 d.	
5	9	$10\frac{2}{3}$	at 1 d.	
L. 170	6	$6\frac{2}{3}$	*Ans.*	

4. A banker in London, with a view to make some little gain, remits to France 6000 crowns, at a time when the rate of exchange was low, viz. at 30 d. Sterling per crown; and when the exchange rose to $31\frac{1}{2}$ d. he drew for the same number of crowns : What did he gain, and how much *per cent.* ?

Cr.

Cr.

8)6000

20) 750 L. paid at 30 d.

37 10, at $1\frac{1}{2}$, equal to the gain.

787 10 Received at $31\frac{1}{2}$ d.

Then ſay,

If 30 : 1.5 : : 100

100

30)150.0(5 *per cent.* *Anſ.*

MORE EXAMPLES.

5. What is the value of 930 crowns 26 ſols 3 deniers, at $32\frac{5}{8}$ d. per crown? *Anſ.* 126 l. 9 s. $7\frac{1}{2}$ d.

6. How many French crowns ſhall I have for 102 l. 13 s. $11\frac{1}{2}$ d. at $31\frac{3}{8}$ d. per crown? *Anſ.* 785 crowns 34 ſols.

France exchanges with other places alſo on the crown, the par being as follows, viz.

1 French crown in		100 French crowns in	
Spain	= $184\frac{3}{8}$ mervadics.	Naples	= $72\frac{1}{4}$ ducats.
Portugal	= $431\frac{4}{5}$ rees.	Florence	= 45 crowns.
Milan	= 62 ſoldi.	Denmark	= 54 rubles.
Sardinia	= $92\frac{1}{2}$ ſoldi.	Sweden	= 78 rixdoll.

IV. *Exchange with Portugal.*

MONEY-TABLE.

		Par in Ster.	*s.*	*d.*	*f.*
		1 ree =	0	0	0.27
400 rees	make	1 cruſade =	2	3	
1000 rees	make	1 millree =	5	$7\frac{1}{2}$	

In Liſbon, Oporto, &c. books and accounts are generally kept in rees and millrees; and the millrees are diſtinguiſhed from the rees by a mark ſet between them thus, 485 ♈ 372; that is, 485 millrees and 372 rees.

Britain,

Britain, as well as other nations, exchanges with Portugal on the millree, the par, as in the table, being $67\frac{1}{2}$ d. Sterling. The courſe with Britain runs from 63 d. to 68 d. Sterling per millree.

EXAMPLES.

1. How much Sterling money will pay a bill of 827 ♈ 160 rees, exchange at $63\frac{3}{8}$ d. Sterling per millree.

$$
\begin{array}{lllll}
 & \textit{Rees.} & \textit{d.} & \textit{Rees.} & \\
\text{If} & 1000 : & 63\frac{3}{8} :: & 827.160 & \\
 & 8 & & 507 & \\
\hline
 & 8000 & 507 & 579012 & \\
 & & & 413580 & \textit{Rem.} \\
\hline
 & & 8000) & 419370.120 & 2 \\
 & & 12) & 52421 \;— & 5\text{ d.} \\
 & & 20) & 4368 \;— & 8\text{ s.} \\
\hline
 & & & \text{L. } 218\ 8\ 5\frac{1}{4} & \textit{Anſ.}
\end{array}
$$

By practice.

$$
\begin{array}{l|l}
 & \textit{Rees.} \\
\textit{d.} & 827.160, \text{ at } 63\frac{3}{8}\text{ d.} \\
\hline
60 = \frac{1}{4} & 206.790 \\
3 = \frac{1}{20} & 10.3395 \\
\frac{2}{8} = \frac{1}{12} & .861625 \\
\frac{1}{8} = \frac{1}{2} & .4308125 \\
\hline
 & 218.4219375
\end{array}
$$

The rees, being thouſandth parts of the millrees, are annexed to the integer, and the operation proceeds exactly as in decimals.

2. How many rees of Portugal will 500 l. Sterling amount to, exchange at 5 s. $4\frac{5}{8}$ d. per millree?

d.

d.	Rees.	L.
If $64\frac{5}{8}$:	1000 : :	500
517	8	20
	8000	10000
		12
		120000
		8000

Rees.
517)960000000(1856.866 *Anſ.*

MORE EXAMPLES.

3. How much Sterling money will pay a bill of 181 millrees, exchange at $64\frac{1}{2}$ d. per millree? *Anſ.* 48 l. 12 s. $10\frac{1}{2}$ d.

4. If I pay in London 933 l. 18 s. 8 d. how many rees ſhall I receive at Liſbon, exchange at 5 s. 4 d. per millree? *Anſ.* 3502.250 rees.

Portugal exchanges with other places as follows, viz. with

		Rees.
Spain, on the piaſtre	=	637
Venice, on the ducat	=	744
Genoa, on the pezzo	=	800
Naples, on the ducat	=	597
Sardinia, on the dollar	=	637
Palermo, on the crown	=	889

V. *Exchange with Spain.*

MONEY-TABLE.

			Par in Ster.	s.	d.
34 mervadies	make	1 rial	=	0	$5\frac{3}{8}$
8 rials	make	1 piaſtre	=	3	7
375 mervadies	make	1 ducat	=	4	$11\frac{1}{4}$

In Madrid, Bilboa, Cadiz, Malaga, Seville, and moſt of the principal places, books and accounts are kept in piaſtres, called alſo *dollars*, rials, and mervadies; and they exchange with Britain generally on the piaſtre, and ſometimes

ſometimes on the ducat. The courſe runs from 35 d. to 45 d. Sterling for a piaſtre or dollar of 8 rials.

EXAMPLES.

1. London imports from Cadiz, goods to the value of 2163 piaſtres and 4 rials: How much Sterling will this amount to, exchange at $38\frac{3}{8}$ d. Sterling per piaſtre?

	Piaſt.	*Rials.*	
	2163	4,	at $38\frac{3}{8}$ d.
$24 = \frac{1}{10}$	216	6	
$12 = \frac{1}{2}$	108	3	
$2 = \frac{1}{6}$	18	0	6
$\frac{2}{8} = \frac{1}{8}$	2	5	$0\frac{3}{4}$
$\frac{1}{8} = \frac{1}{2}$	1	2	$6\frac{3}{8}$
	345	17	$1\frac{1}{8}$
		1	$7\frac{3}{16}$
L.	345	18	$8\frac{5}{16}$ *Anſ.*

	d.
	$38\frac{3}{8}$ each.
Rials 4 =	$19\frac{3}{16}$

2. London remits to Cadiz 345 l. 18 s. $8\frac{5}{16}$ d. How much Spaniſh money will this amount to, exchange at $38\frac{3}{8}$ d. Sterling per piaſtre?

	d.		*Piaſt.*		*L.*	*s.*	*d.*
If	$38\frac{3}{8}$	:	1	::	345	18	$8\frac{5}{16}$

```
 307          20         614)1328389(2163 piaſtres.
   2        ----              1228...
 ---        6918              -------
 614          12               1003
           -----                614
           83024               ----
              16               3898
          ------               3684
          498149               ----
           83024               2149
         -------               1842
         1328389               ----
                                307
                                  8
                         614)2456(4 rials.
                             2456
```

MORE

MORE EXAMPLES.

3. Spain remits to Britain 1768 dollars, or piaſtres, 7 rials, at $40\frac{7}{8}$ d. Sterling per piaſtre: How much Sterling will this amount to? *Anſ.* L. 301 : 5 : $2\frac{5}{8}$.

4. If I pay in Britain L. 1000 : 16 : 10, how much Spaniſh money may I draw for, exchange at $41\frac{1}{2}$ d. Sterling per piaſtre? *Anſ.* 5788 piaſtres.

The ſilver money in Spain is of two ſorts, viz. old and new plate; the old plate is 25 *per cent.* better than the new; and ſo the piaſtre of exchange conſiſts of 8 rials old plate, or of 10 rials new plate.

The copper money in Spain is called *vellon*; and though the piaſtre vellon be of the ſame value with the piaſtre of exchange old plate, yet the rial vellon is little more than half a rial old plate; for 32 rials vellon are equal in value to 17 rials old plate, or 100 rials vellon are equal to $53\frac{1}{8}$ rials old plate.

Spain exchanges with other places as follows, viz. with

		Mervadies.
Venice, on the ducat,	=	318
Genoa, on the pezzo,	=	$341\frac{5}{8}$
Rome, on the crown,	=	464
Florence, on the crown,	=	409
Naples, on the ducat,	=	255
Milan, on the ducat,	=	348
Bologna, on the dollar,	=	$322\frac{5}{8}$
Meſſina, on the crown,	=	380

VI. *Exchange with Venice.*

MONEY-TABLE.

$5\frac{1}{6}$ Soldi } make { 1 gros
24 Gros } make { 1 ducat = $50\frac{1}{4}$ d. Sterling.

The money of Venice is of three ſorts, viz. two of bank money, and the picoli money. One of the banks

deals in banco money, and the other in banco current. The bank money is 20 *per cent.* better than the banco current, and the banco current 20 *per cent.* better than the picoli money. Exchanges are always negotiated by the ducat banco, the par being 4 s. $2\frac{1}{4}$ d. Sterling, as in the table.

Though the ducat be commonly divided into 24 gros, yet bankers and negotiators, for facility of computation, usually divide it as follows, and keep their books and accounts accordingly.

12 Deniers d'or	make	1 sol d'or
20 Sols d'or		1 ducat = $50\frac{1}{4}$ d. Sterling.

The course of exchange is from 45 d. to 55 d. Sterling per ducat.

EXAMPLES.

1. How much Sterling money is equal to 1459 ducats 18 sols 1 denier, bank money of Venice, exchange at $52\frac{3}{4}$ d. Sterling per ducat?

Duc. *d.* *Duc.* *sol.* *den.*

If 1 : $52\frac{3}{4}$ :: 1459 18 1

$52\frac{3}{4}$

2918

7295

d. 75868

$\frac{1}{2} = 729\frac{4}{8}$

$\frac{1}{4} = 364\frac{6}{8}$

$76962\frac{2}{8}$

$47\frac{5}{8}$ *Rem.*

12)77010(6 d.

2|0)641|7(17 s.

L. 320 17 6 Sterling. *Ans.*

Sols.		*d.*
		$52\frac{3}{4}$ rate.
10	$= \frac{1}{2}$	$26\frac{3}{8}$
5	$= \frac{1}{2}$	$13\frac{1}{8}$
2	$= \frac{1}{5}$	$\frac{2}{8}$
1	$= \frac{1}{2}$	$\frac{5}{8}$
den. 1	$= \frac{1}{12}$	$0\frac{2}{8}$
		$47\frac{5}{8}$

2. How

2. How many ducats at Venice are equal to 385 l. 12 s. 6 d. Sterling, exchange at 4 s. 4 d. per ducat?

```
     L.     Duc.   L.
If .21ɞ : 1 :: 385.625

          .21ɞ)385.625
           21  385625     Duc.
          195)347062.5(1779.8 Anſ.
              195
              1520
              1365
               1556
               1365
                1912
                1755
                 1575
                 1560
                 (15)
```

Bank money is reduced to current money, by allowing for the agio, as was done in exchange with Holland; viz. ſay, As 100 to 120, or as 10 to 12, or as 5 to 6, ſo the given bank money to the current ſought. And current money is reduced to bank money by reverſing the operation. And in like manner may picoli money be reduced to current or to bank money, and the contrary.

100 ducats banco of Venice.

In Leghorn	= 93 pezzos	In Lucca	= 77 crowns
In Rome	= $68\frac{1}{2}$ crowns	In Frankfort	= $139\frac{1}{2}$ florins

VII. *Exchange with Genoa.*

MONEY-TABLE.

				s.	d.	
12 Denari	make	1 ſoldi				
20 Soldi	make	1 pezzo	=	4	6	Sterling.

Books and accounts are generally kept in pezzos, ſoldi, and denari; but ſome keep them in lires, ſoldi, and denari; and 12 ſuch denari make 1 ſoldi, and 20 ſoldi make 1 lire.

The pezzo of exchange is equal to $5\frac{3}{4}$ lires; and, conſequently, exchange money is $5\frac{3}{4}$ times better than the lire money. The courſe of exchange runs from 47 d. to 58 d. Sterling per pezzo.

EXAMPLE.

How much Sterling money is equivalent to 3390 pezzos 16 ſoldi, of Genoa, exchange at $51\frac{7}{8}$ d. Sterling per pezzo?

	Soldi.	d.		Pez.	ſoldi.
If	20 :	$51\frac{7}{8}$	: :	3390	16
	8				20
	160	415		67816	
				415	
				339080	
				67816	
				271264	

$$160)28143640(175897\tfrac{3}{4} \text{ d.} = 732 \text{ L. } 18 \text{ s. } 1\tfrac{3}{4} \text{ d.}$$

If Sterling money be given, it may be reduced or changed into pezzos of Genoa, by reverſing the former operation.

Exchange money is reduced to lire money, by being multiplied by $5\frac{3}{4}$, as follows.

	Pez.	ſoldi.	Decimally.
	3390	16	3390.8
		$5\frac{3}{4}$	5.75
	16954	0	169540
$\frac{1}{2}$ =	1695	8	237356
$\frac{1}{4}$ =	847	14	169540
Lires	19497	2	Lires 19497.100

And

And lire money is reduced to exchange money by dividing it by $5\frac{3}{4}$.

			Soldi of Genoa.
In Milan,	1 crown	=	80
In Naples,	1 ducat	=	86
In Leghorn,	1 piastre	=	20
In Sicily,	1 crown	=	$127\frac{2}{9}$

VIII. *Exchange with Leghorn.*

MONEY-TABLE.

				s.	d.	
12 Denari	make	1 soldi				
20 Soldi	make	1 piastre	=	4	6	*Ster.*

Books and accounts are kept in piastres, soldi, and denari. The piastre here consists of 6 lires, and the lire contains 20 soldi, and the soldi 12 denari, and consequently exchange money is 6 times better than lire money. The course of exchange is from 47 d. to 58 d. Sterling per piastre.

EXAMPLE.

What is the Sterling value of 731 piastres, at $55\frac{1}{2}$ d. each?

s.	d.				
			731 piastres, at $55\frac{1}{2}$ d.		
4 or	48	$=\frac{1}{5}$	146	4	
	6	$=\frac{1}{8}$	18	5	6
	$1\frac{1}{2}$	$=\frac{1}{4}$	4	11	$4\frac{1}{2}$
		L.	169	0	$10\frac{1}{2}$ *Ans.*

Sterling money is reduced to money of Leghorn, by reversing the former operation; and exchange money is reduced to lire money by multiplying by 6, and lire money to exchange money by dividing by 6.

100 piastres of Leghorn are
In Naples = 134 ducats. | In Geneva = $185\frac{1}{3}$ crowns.

Soldi of Leghorn.
In Sicily, 1 crown = $133\frac{1}{3}$
In Sardinia, 1 dollar = $95\frac{5}{9}$

The above are the chief places in Europe with which Britain exchanges directly; the exchanges with other places are generally made by bills on Hamburgh, Holland, or Venice. I shall here however subjoin the par of exchange betwixt Britain and most of the other places in Europe, with which we have any commercial intercourse.

	Par in Sterling.			L.	s.	d.
Rome,	1	crown	=		6	$1\frac{2}{3}$
Naples,	1	ducat	=		3	$4\frac{1}{2}$
Florence,	1	crown	=		5	$4\frac{5}{8}$
Milan,	1	ducat	=		4	7
Bologna,	1	dollar	=		4	3
Sicily,	1	crown	=		5	0
Vienna,	1	rixdollar	=		4	8
Ausburgh,	1	florin	=		3	$1\frac{1}{3}$
Francfort,	1	florin	=		3	0
Bremen,	1	rixdollar	=		3	6
Breslau,	1	rixdollar	=		3	3
Berlin,	1	rixdollar	=		4	0
Stetin,	1	mark	=		1	6
Emden,	1	rixdollar	=		3	6
Bolsenna,	1	rixdollar	=		3	8
Dantzic,	$13\frac{1}{2}$	florins	=	1	0	0
Stockholm,	$34\frac{4}{7}$	dollars	=	1	0	0
Russia,	1	rubble	=		4	5
Turkey,	1	asper	=		4	6

The following places, viz. Switzerland, Noremburgh, Leipsic, Dresden, Osnaburgh, Brunswic, Cologn, Liege, Strasburgh, Cracow, Denmark, Norway, Riga, Revel, Narva, exchange with Britain, when direct exchange is made, upon the rixdollar, the par being 4 s. 6 d. Sterling.

IX. *Ex-*

IX. *Exchange with America and the West Indies.*

In North America and the West Indies, accounts, as in Britain, are kept in pounds, shillings, and pence. In North America they have few coins circulating among them, and on that account have been obliged to substitute a paper-currency for a medium of their commerce; which having no intrinsic value, is subjected to many disadvantages, and generally suffers a great discount. In the West Indies coins are more frequent, owing to their commercial intercourse with the Spanish settlements.

Exchange betwixt Britain and America, or the West Indies, may be computed as in the following examples.

1. The neat proceeds of a cargo from Britain to Boston amount to 845 l. 17 s. 6 d. currency: How much is that in Sterling money, exchange at 80 *per cent.*?

If 180	:	100			
18	:	10	*L.*	*s.*	*d.*
9	:	5 ::	845	17	6
					5
		9)	4229	7	6
		L.	469	18	$7\frac{1}{3}$ Ster. *Ans.*

2. Boston remits to Britain a bill of 469 l. 18 s. $7\frac{1}{3}$ d. Sterling: How much currency was paid for the bill at Boston, exchange at 80 *per cent.*?

If 100	:	180	*L.*	*s.*	*d.*
5	:	9 ::	469	18	$7\frac{1}{3}$
					9
		5)	4229	7	6
		L.	845	17	6 currency. *Ans.*

3. Philadelphia is indebted to Britain in 985 l. 12 s. 3 d. currency: How much Sterling will that amount to, exchange at 75 *per cent.*?

If

If 175 : 100				
35 : 20		*L.*	*s.*	*d.*
7 : 4 ::		985	12	3
				4
	7)	3942		9
	L.	563	4	$1\frac{5}{7}$ Ster. *Anſ.*

4. Philadelphia grants a bill on Britain for 563 l. 4 s. $1\frac{5}{7}$ d. Sterling: What is the value of the bill in currency, exchange at 75 *per cent.*?

If 100 : 175		*L.*	*s.*	*d.*
4 : 7 ::		563	4	$1\frac{5}{7}$
				7
	4)	3942		9
	L.	985	12	3 currency, *Anſ.*

By practice thus:

	L.	*s.*	*d.*
100 =	563	4	$1\frac{5}{7}$
50 = $\frac{1}{2}$	281	12	$0\frac{6}{7}$
25 = $\frac{1}{2}$	140	16	$0\frac{3}{7}$
175	985	12	3 currency.

5. A parcel of goods from Britain, amounting, by invoice, to 323 l. 5 s. Sterling, are ſold by a factor in Virginia for L. 469 : 2 : 6 currency: What Sterling ought the factor to remit, deducing 5 *per cent.* for commiſſion? and how much does the employer gain *per cent.* when exchange is at 30 *per cent.*?

Exch.	*com.*		
If 130 + 5 = 135 : 100		L.	
27 :	20 : :	469.125	
		20	
		3)9382.500	
		9)3127 5	
		347.5	Ster. to be remitted.
		323.25	value consigned.
		24.25	gain.

If 323.25 : 24.25 : : 100
100

323.25)2425.00(7.5 or $7\frac{1}{2}$ *per cent.*
226275
162250
161625
625

6. How much currency in Virginia will purchase a bill of 345 l. 13 s. 4 d. Sterling, when exchange is $33\frac{1}{3}$ *per cent.* ?

If 100 : $133\frac{1}{3}$
3

300 : 400

	L.	s.	d.
3 : 4 : :	345	13	4
			4
3)	1382	13	4
Anf. currency	460	17	$9\frac{1}{3}$

By practice.

100 =	345	13	4	
$33\frac{1}{3} = \frac{1}{3}$	115	4	$5\frac{1}{3}$	
$133\frac{1}{3}$	460	17	$9\frac{1}{3}$	cur.

7. If an ounce of gold worth L. 4 Sterling be rated in Carolina at L. 9 : 15 : 7 currency, how much currency will L. 100 Sterling purchase ?

```
   L.  currency.    L.
If 4 : 9  15  7 : : 100
   1          12        25 = 12 + 12 + 1
   -------------
     117   7
     117   7
       9  15  7
   -------------
L. 244   9  7 currency.   Anſ.
```

8. How much Sterling money will 1780 l. Jamaica currency amount to, exchange at 40 *per cent.*?

```
If 140 : 100
    14 : 10       L.
     7 :  5 : : 1780
                   5
                ----
              7)8900   s. d.
                ----
                1271   8 6¾ Ster.  Anſ.
```

Bills of exchange from America, the rate being high, is an expenſive way of remitting money to Britain; and therefore merchants in Britain generally chuſe to have the debts due to them remitted home in ſugar, rum, or other produce.

X. *Exchange with Ireland.*

At Dublin, and all over Ireland, books and accounts are kept in pounds, ſhillings, and pence, as in Britain; and they exchange on the 100 l. Sterling.

The par of one ſhilling Sterling is one ſhilling and one penny Iriſh; and ſo the par of 100 l. Sterling is 108 l. 6 s. 8 d. Iriſh. The courſe of exchange runs from 6 to 15 *per cent.*

EXAMPLES.

1. London remits to Dublin 586 l. 10 s. Sterling: How much Iriſh money will that amount to, exchange at $9\frac{5}{8}$ *per cent.*?

If

$$\begin{array}{llll} & & & \text{L.} \\ \text{If } 100 & : 109\tfrac{5}{8} & :: & 586.5 \\ 8 & & & 877 \\ \hline 800 & : 877 & & 41055 \\ & & & 41055 \\ & & & 46920 \\ \hline & & 800) & 514360.5 \\ \hline & & & 642.950625 \end{array}$$

Anſ. 642 l. 19 s. Iriſh.

By practice.

$$\begin{array}{l|l} \textit{p. cent.} & 586.5 \\ 10 = \frac{1}{10} & 58.65 \\ 2 = \frac{1}{5} & 11.73 \text{ ſub.} \\ \hline 8 = & 46.92 \\ 1 = \frac{1}{8} & 5.865 \\ \frac{4}{8} = \frac{1}{2} & 2.9325 \\ \frac{1}{8} = \frac{1}{4} & .733125 \\ \hline 9\frac{5}{8} & 56.450625 \text{ add.} \\ & 642.950625 \end{array}$$

2. How much Sterling will 625 l. Iriſh amount to, exchange at $10\frac{3}{8}$ *per cent.*?

$$\begin{array}{llll} \text{If } 110\tfrac{3}{8} : & 100 & :: & 625 \\ & 8 & & 800 \quad \text{L.} \quad \text{s.} \quad \text{d.} \\ \hline 883 & 800 & & 883)500000(566 \quad 5 \quad 0\tfrac{1}{4} \text{ Ster. } \textit{Anſ.} \end{array}$$

3. Britain remitted to Ireland 10000 l. Sterling when exchange was at 12 *per cent.*; but exchange falling to 8 *per cent.* Britain draws back the whole remittance: How much Sterling was drawn back, and how much did Britain gain?

As 108 : 112 :: 10000

```
            10000     L.    s.  d.
108)1120000(10370    7   4¾ Ster. drawn back.
             10000
             ----------------
               370    7   4¾ gained.
```

Which is more than $3\frac{1}{2}$ *per cent.*

4. When exchange was up at 12 *per cent.* Dublin drew on London for 10000 l. Sterling; and when exchange fell to 8 *per cent.* Dublin remitted 10000 l. Sterling to London: How much Irish money did Dublin gain?

```
          Irish.
           112
           108
  Ster.    ---     Ster.
If 100  :   4  ::  10000
               10000
               -----
      100)  40000(400l. Irish gained.
```

XI. *Exchange betwixt London and other places in Britain.*

The several towns in Britain exchange with London for a small premium in favour of London; such as, 1, $1\frac{1}{4}$, *&c. per cent.* The premium is more or less according to the demand for bills.

EXAMPLE.

Edinburgh draws on London for 860 l. exchange at $1\frac{3}{8}$ *per cent.*: How much money must be paid at Edinburgh for the bill?

per cent.	L.			
	860			
$1 = \frac{1}{100}$	8	12		
$\frac{2}{8} = \frac{1}{4}$	2	3		
$\frac{1}{8} = \frac{1}{2}$	1	1	6	
	11	16	6	premium.
	871	16	6	paid for the bill.

To avoid paying the premium, it is an uſual practice to take the bill payable at London a certain number of days after date; and in this way of doing, 73 days is equivalent to 1 *per cent.*

XII. *Arbitration of Exchanges.*

The courſe of exchange betwixt nation and nation naturally riſes or falls according as the circumſtances and balance of trade happen to vary. Now, to draw upon, and remit to foreign places, in this fluctuating ſtate of exchange, in the way that will turn out moſt profitable, is the deſign of arbitration. Which is either ſimple or compound.

I. *Simple arbitration.*

In ſimple arbitration, the rates or prices of exchange from one place to other two are given; whereby is found the correſpondent price between the ſaid two places, called the *arbitrated price*, or *par of arbitration*: and hence is derived a method of drawing and remitting to the beſt advantage.

EXAMPLES.

1. If exchange from London to Amſterdam be 33 s. 9 d. per l. Sterling, and if exchange from London to Paris be 32 d. per crown, what muſt be the rate of exchange from Amſterdam to Paris, in order to be on a par with the other two?

Ster.

```
  Ster.  Flem.   Ster.
   s.    s. d.    d.
If 20 : 33  9 : : 32
   12   12
  ---   ---
  240  405
        32
      -----
       810
     1215
     -----
240)12960(54 d. Flem. per crown.   Anſ.
```

2. If exchange from Paris to London be 32 d. Sterling per crown; and if exchange from Paris to Amſterdam be 54 d. Flemiſh per crown; what muſt be the rate of exchange between London and Amſterdam, in order to be on a par with the other two?

```
  Ster.  Flem.  Ster.
    d.    d.     d.
If 32  :  54 : : 240
         240
         ---
         216
       108
       --------12)    s.  d.
   32)12960 (405 (33  9 Flem. per l. Ster.   Anſ.
```

3. If exchange from Amſterdam to Paris be 54 d. Flem. per crown, and if exchange from Amſterdam to London be 33 s. 9 d. Flem. per l. Sterling; what muſt be the rate of exchange between Paris and London, in order to be on a par with the other two?

```
   Flem.   Ster.   Flem.
   s. d.    d.      d.
If 33  9 : 240 : :  54
    12      54
  -----    ---
   405      96
         120
         -----
   405)12960(32 d. Ster. per crown.   Anſ.
```

From

From these three operations, it appears, that if any sum of money be remitted, at the rates of exchange mentioned, from any one of the three places to the second, and from the second to the third, and again from the third to the first, the sum so remitted will come home entire, without increase or diminution.

From the par of arbitration thus found, and the course of exchange given, is deduced a method of drawing and remitting to advantage, as in the following examples.

4. If exchange from London to Paris be 32 d. Sterling per crown, and to Amsterdam 405 d. Flemish per l. Sterling; and if, by advice from Holland or France, the course of exchange between Paris and Amsterdam is fallen to 52 d. Flemish per crown; what may be gained *per cent.* by drawing on Paris, and remitting to Amsterdam?

The par of arbitration between Paris and Amsterdam in this case, by Ex. 1. is 54 d. Flemish per crown. Work as under.

	d. St.		*Cr.*		*L. St.*		*Cr.*			
If	32	:	1	::	100	:	750 debit at Paris.			

	Cr.		*d. Fl.*		*Cr.*		*d. Fl.*			
If	1	:	52	::	750	:	39000 credit at Amsterdam.			

	d. Fl.		*L. St.*		*d. Fl.*		*L.*	*s.*	*d. Ster.*	
If	405	:	1	::	39000	:	96	5	$11\frac{1}{9}$	to be remitted.
							100			
							3	14	$0\frac{8}{9}$	gained *per cent.*

But if the course of exchange between Paris and Amsterdam, instead of falling below, rise above the par of arbitration, suppose to 56 d. Flemish per crown; in this case, if you propose to gain by the negotiation, you must draw on Amsterdam, and remit to Paris. The computation follows.

L.

	L.St. d.Fl.	L.St.	d. Fl.	
If	1 : 405 ::	100 :	40500	debit at Amſterdam.
	d.Fl. Cr.	d. Fl.	Cr.	
If	56 : 1 ::	40500 :	$723\frac{3}{14}$	credit at Paris.
	Cr. d.St.	Cr.	L. s. d. Ster.	
If	1 : 32 ::	$723\frac{3}{14}$:	96 8 $6\frac{6}{7}$	to be remitted.
			100	
			3 11 $5\frac{1}{7}$	gained *per cent.*

In negotiations of this ſort, a fund for remittance is afforded out of the ſum you receive for the draught; and your credit at the one foreign place pays your debit at the other.

5. London is indebted to Peterſburgh 5000 rubles, and Peterſburgh can draw for them directly on England at 50 d. Sterling per ruble, or on Holland at 90 d. Flemiſh per ruble: Which of theſe two ways will be moſt advantageous to London, ſuppoſing the courſe of exchange between London and Holland to be 36 s. 4 d. Flemiſh per l. Sterling?

Find the par of arbitration, by ſaying,

	d.St. d.Fl.	d. St.	d. Fl.	
If	50 : 90 ::	240 :	432	or 36 s. Flemiſh.

If the courſe of exchange between London and Holland had been the ſame with the par of arbitration, viz. 36 s. payment in either way would be the ſame to London; but ſince the courſe is 36 s. 4 d. it is plainly an advantage to make payment by the way of Holland; for it is better to receive 36 s. 4 d. than 36 s. for a pound Sterling. The computation follows.

	Rub. d. St.	Rub.	L. s. d. Ster.	
If	1 : 50 ::	5000 :	1041 13 4	Lond. to Peterſburgh.
	Rub. d. Fl.	Rub.	d. Fl.	
If	1 : 90 ::	5000 :	450000	Holland to Peterſburgh.
	s. d.F. L.St. d. Fl.		L. s. d. Ster.	
If	36 4 : 1 ::	450000 :	1032 2 $2\frac{1}{4}$	Lond. to Holland.
			9 11 $1\frac{3}{4}$	gain.

If

If the courſe of exchange to Holland had fallen below 36 s. the direct exchange would have been preferable.

The following, or like queſtions, are ſolved, without finding the par of arbitration.

6. London was ordered to remit 500 ducats to Venice, at 50 d. Sterling per ducat, and to draw for the value upon Spain, at 40 d. Sterling per piaſtre; but when the order came to hand, bills on Venice were at $52\frac{1}{2}$ d.: At what rate of exchange muſt London draw upon Spain, to compenſate the advance upon the remittance?

$$\begin{array}{lccccc} & \text{d.} & & \text{d.} & & \text{d.} \\ \text{If} & 50 & : & 52\frac{1}{2} & :: & 40 \end{array}$$

$$\begin{array}{lccccc} \text{Or, If} & 100 & : & 105 & :: & 40 \\ & & & 40 & & \\ \hline \end{array}$$

$$100)4200(42 \text{ d. } \textit{Anſ.}$$

Proof.

$$\begin{array}{lccccccc} & \text{Duc.} & & \text{d. St.} & & \text{Duc.} & & \text{d. St.} \\ \text{If} & 1 & : & 50 & :: & 500 & : & 25000 \end{array}$$

$$\begin{array}{lccccccc} & \text{d.} & & \text{Piaſt.} & & \text{d.} & & \text{Piaſt.} \\ \text{If} & 40 & : & 1 & :: & 25000 & : & 625 \end{array}$$

Then, by the courſe,

$$\begin{array}{lccccccc} & \text{Duc.} & & \text{d.} & & \text{Duc.} & & \text{d.} \\ \text{If} & 1 & : & 52\frac{1}{2} & :: & 500 & : & 26250 \end{array}$$

$$\begin{array}{lccccccc} & \text{d.} & & \text{Piaſt.} & & \text{d.} & & \text{Piaſt.} \\ \text{If} & 42 & : & 1 & :: & 26250 & : & 625 \text{ as before.} \end{array}$$

If the courſe of exchange with Spain be lower than 42 d. the order cannot be executed without loſs; and if it be higher, there will be gain.

7. London was ordered to remit to Paris 800 crowns, at $31\frac{3}{8}$ d. Sterling per crown, and to draw for

 the

the value upon Amſterdam, at 36 s. 9 d. Flem. per pound Ster.; but when the order came up, bills on Paris were at $31\frac{5}{8}$ d. Sterling per crown: What muſt be the rate of exchange with Amſterdam to compenſate the advance on the remittance?

	d.		s. d.		d.		s. d.	
If	$31\frac{5}{8}$	:	36 9	::	$31\frac{3}{8}$	:	36 $5\frac{1}{2}$	nearly *Anſ.*

The proportion is inverſe; for the rate of exchange with Holland muſt be leſſened to anſwer our purpoſe; which may be proved as in the former example. The proportions for the remittance and draught follow.

	Cr.		d. St.		Cr.		d. Ster.
If	1	:	$31\frac{5}{8}$	::	800	:	25300

	d. St.		s. d. Fl.		d. St.		d. Fl.
If	240	:	36 $5\frac{1}{2}$	::	25300	:	$46119\frac{1}{2}$ nearly.

8. A merchant in London had 6000 guilders in the bank at Amſterdam, and was offered 22 d. Sterling for each guilder; but as this offer did not pleaſe, he indorſes a bill for the whole to his factor at Paris; who ſoon brought the money to France, by exchanging at 55 d. Flemiſh per crown: he allows the factor $\frac{1}{2}$ *per cent.* commiſſion for his trouble, and then draws upon him for the whole, exchange at 32 d. Sterling per crown: How much was this better than the offer of 22 d. per guilder?

6000 guilders.
40 d. Flem. in 1 guilder.

	d. Fl.		Cr.				
If	55	:	1	::	240000	:	4363.63, crowns.
							21.81, com. = $\frac{1}{200}$
							4341.81, neat.

Cr.

Cr. d. Cr.					
If 1 : 32 :: 4341.81,					
32					
8683,63,					
130254,54,		L.	s.	d. Ster.	
138938.18,	=	578	18	2	by exch.
		550	0	0	by offer.
Guild. 6000, at 22d. St.		28	18	2	gained.

d.		Guild.
		6000, at 22d. St.
20	= $\frac{1}{12}$	500
2	= $\frac{1}{10}$	50
		550 l. Ster.

II. *Compound arbitration.*

In compound arbitration, the rate or price of exchange between three, four, or more places, is given, in order to find how much a remittance paſſing through them all will amount to at the laſt place; or to find the arbitrated price, or par of arbitration, between the firſt place and the laſt. And this may be done by the following

RULES.

I. Diſtinguiſh the given rates or prices into antecedents and conſequents; place the antecedents in one column, and the conſequents in another on the right, fronting one another by way of equation.

II. The firſt antecedent, and the laſt conſequent to which an antecedent is required, muſt always be of the ſame kind.

III. The ſecond antecedent muſt be of the ſame kind with the firſt conſequent, and the third antecedent of the ſame kind with the ſecond conſequent, &c.

IV. If to any of the numbers a fraction be annexed, both the antecedent and its conſequent muſt be multiplied into the denominator.

V. To facilitate the operation, terms that happen to be

be equal or the same in both columns, may be dropped or rejected, and other terms may be abridged.

VI. Multiply the antecedents continually for a divisor, and the consequents continually for a dividend, and the quot will be the answer, or antecedent required.

EXAMPLES.

1. If London remit 1000 l. Sterling to Spain, by way of Holland, at 35 s. Flemish per l. Sterling; thence to France, at 58 d. Flemish per crown; thence to Venice, at 100 crowns per 60 ducats; and thence to Spain, at 360 mervadies per ducat; how many piastres, of 272 mervadies, will the 1000 l. Sterling amount to in Spain?

Antecedents.		*Consequents.*	*Abridged.*		
1 l. Sterling	=	35 s. or 420 d. Fl.	1	=	210
58 d. Flemish	=	1 crown France	29	=	1
100 crowns France	=	60 ducats Venice	1	=	30
1 ducat Venice	=	360 mervadies Spain	1	=	45
272 mervadies	=	1 piastre	17	=	1
How many piastres	=	1000 l. Sterling?		=	10

In order to abridge the terms, divide 58 and 420 by 2, and you have the new antecedent 29, and the new consequent 210; reject two ciphers in 100 and 1000; divide 272 and 360 by 8, and you have 34 and 45; divide 34 and 60 by 2, and you have 17 and 30; and the whole will stand abridged as above.

Then, $29 \times 17 = 493$ divisor; and $210 \times 30 \times 45 \times 10 = 2835000$ dividend; and 493)2835000($5750\frac{1}{2}$ piastres. *Ans.*

Or, the consequents may be connected with the sign of multiplication, and placed over a line by way of numerator, and the antecedents, connected in the same manner, may be placed under the line, by way of denominator, and then abridged, as follows.

$$\frac{420 \times 60 \times 360 \times 1000}{58 \times 100 \times 272} = \frac{210 \times 60 \times 360 \times 10}{29 \times 1 \times 272} = \frac{210 \times 60 \times 45 \times 10}{29 \times 34}$$

$$= \frac{210 \times 30 \times 45 \times 10}{29 \times 17} = \frac{2835000}{493},$$

And, 493)2835000(5750$\frac{1}{2}$ piastres. *Ans.*

The placing the terms by way of antecedent and consequent, and working as the rules direct, saves so many statings of the rule of three, and greatly shortens the operation. The proportions at large for the above question would stand as under.

	L.St.	*d. Fl.*		*L. St.*		*d. Fl.*
If	1	: 420	::	1000	:	420000
	d. Fl.	*Cr.*		*d. Fl.*		*Cr.*
If	58	: 1	::	420000	:	$7241\frac{11}{29}$
	Cr.	*Duc.*		*Cr.*		*Duc.*
If	100	: 60	::	$7241\frac{11}{29}$	:	$4344\frac{24}{29}$
	Duc.	*Mer.*		*Duc.*		*Mer.*
If	1	: 360	::	$4344\frac{24}{29}$	:	$1564137\frac{27}{29}$
	Mer.	*Piast.*		*Mer.*		*Piast.*
If	272	: 1	::	$1564137\frac{27}{29}$	:	$5750\frac{250}{493}$

If we suppose the course of direct exchange to Spain to be 42$\frac{1}{2}$ d. Sterling per piastre, the 1000 l. remitted would only amount to 5647$\frac{1}{2}$ piastres; and, consequently, 103 piastres are gained by the negotiation; that is, about 2 *per cent.*

2. A banker in Amsterdam remits to London 400 l. Flemish; first to France at 56 d. Flemish per crown; from France to Venice at 100 crowns per 60 ducats; from Venice to Hamburgh at 100 d. Flemish per ducat; from Hamburgh to Lisbon at 50 d. Flemish per crusade of 400 rees; and, lastly, from Lisbon to London at 64 d. Sterling per millree: How much Sterling money will the remittance amount to? and how much will be gained or saved, supposing the direct exchange from Holland

Holland to London at 36 s. 10 d. Flemish per l. Sterling?

Antecedents.		*Consequents.*
56 d. Flem.	=	1 crown.
100 crowns	=	60 ducats.
1 ducat	=	100 d. Flem.
50 d. Flem.	=	400 rees.
1000 rees	=	64 d. Sterling.
How many d. Ster.	=	400 l. or 96000 d. Flemish?

This, in the fractional form, will stand as follows.

$$\frac{60 \times 100 \times 400 \times 64 \times 96000}{56 \times 100 \times 50 \times 1000} = \frac{368640}{7}, \text{ and}$$

7)368640(52662$\frac{6}{7}$ d. Ster. = 219 l. 8 s. 6$\frac{6}{7}$ d. St. *Ans.*

To find how much the exchange from Amsterdam directly to London, at 36 s. 10 d. Flemish per l. Sterling will amount to, say,

s.	*d*		*d. Fl.*		*L. St.*		*d. Fl.*		*L.*	*s.*	*d. St.*
36	10	If	442	:	1	::	96000	:	217	3	10$\frac{1}{2}$
12									219	8	6$\frac{3}{4}$
442			Gained or saved,						2	4	8$\frac{1}{4}$

In the above example, the par of arbitration, or the arbitrated price, between London and Amsterdam, viz. the number of Flemish pence given for 1 l. Sterling, may be found thus,

Make 64 d. Sterling, the price of the millree, the first antecedent; then all the former consequents will become antecedents, and all the antecedents will become consequents. Place 240, the pence in 1 l. Sterling, as the last consequent, and then proceed as taught above, viz.

Antecedents

Antecedents.		*Consequents.*
64 d. Ster.	=	1000 rees.
400 rees	=	50 d. Flem.
100 d. Flem.	=	1 ducat.
60 ducats	=	100 crowns.
1 crown	=	56 d. Flem.
How many d. Flem.	=	240 d. Ster.?

$$\frac{1000 \times 50 \times 100 \times 56 \times 240}{64 \times 400 \times 100 \times 60} = \frac{875}{2}, \text{ and}$$

2)875(437½ d. = 36 s. 5½ d. Flem. per l. Ster. *Anſ.*

Or the arbitrated price may be found from the anſwer to the queſtion, by ſaying,

	d Ster.		*d. Flem.*		*d. St.*
If	$\frac{368640}{7}$	:	96000	: :	240

```
             96000
                 7
            ------
            672000
               240
          --------
             2688
            1344
            --------     d.       s.  d. Flem.
368640)161280000(437½ = 36  5½ as before.
```

The work may be proved by the arbitrated price thus: As 1 l. Sterling to 36 s. 5½ d. Flemiſh, ſo 219 l. 8 s. $6\frac{6}{7}$ d. Sterling to 400 l. Flemiſh.

The arbitrated price compared with the direct courſe ſhows whether the direct or circular remittance will be moſt advantageous, and how much. Thus the banker at Amſterdam will think it better exchange to receive 1 l. Sterling for 36 s. 5½ d. Flemiſh, than for 36 s. 10 d. Flemiſh.

3. A merchant at London has credit for 680 piaſtres at Leghorn, for which he can draw directly at 50 d. Sterling per piaſtre; but chuſing to try the circular way, they are by his order remitted, firſt to Venice at 94 piaſtres per 100 ducats banco; thence to Cadiz at 320 mervadies per ducat; thence to Liſbon at 630 rees per piaſtre

piaſtre of 272 mervadies; thence to Amſterdam at 50 d. Flemiſh per cruſade of 400 rees; thence to Paris at 56 d. Flemiſh per crown; thence to London at $31\frac{1}{3}$ d. Sterling per crown: What is the arbitrated price between London and Leghorn per piaſtre? and how much is the circular remittance better than the direct draught, without reckoning charges?

Antecedents.		*Conſequents.*
94 piaſtres	=	100 ducats.
1 ducat	=	320 mervadies.
272 mervad.	=	630 rees.
400 rees.	=	50 d. Flem.
56 d. Flem.	=	1 crown.
3 crowns	=	94 d. St. = $3 \times 31\frac{1}{3}$
How many d. Ster.	=	1 piaſtre?

$$\frac{100 \times 320 \times 630 \times 50 \times 94}{94 \times 272 \times 400 \times 56 \times 3} = \frac{1875}{34}, \text{ and}$$

34)1875($55\frac{5}{34}$ d. Sterling per piaſtre. *Anſ.*

			L.	*s.*	*d.*
680 piaſtres,	at $55\frac{5}{34}$ d.	=	156	5	0
———	at 50 d.	=	141	13	4
	Gained		14	11	8

If the arbitrated price of any intermediate antecedent be required, put the queſtion at the void place, fill up the place of the firſt queſtion with its anſwer; or, to avoid the fraction, make 680 piaſtres the laſt conſequent, and their value in pence Sterling the laſt antecedent, and then work as before.

Ex. How many rees are equal in value to 50 d. Flemiſh?

Antecedents.

Antecedents.		*Consequents.*
94 piastres	=	100 ducats.
1 ducat	=	320 mervadies.
272 mervad.	=	630 rees.
How many rees	=	50 d. Flem.?
56 d. Flem.	=	1 crown.
3 crowns	=	94 d. Sterling.
37500 d. Ster.	=	680 piastres.

$$\frac{100 \times 320 \times 630 \times 50 \times 94 \times 680}{94 \times 272 \times 56 \times 3 \times 37500} = \frac{5 \times 2 \times 10 \times 4}{1} = 400$$

rees. *Anſ.*

If the arbitrated price of a conſequent be required, at the void place put the queſtion, and work as before; only in this caſe the product of the antecedents is the dividend, and the product of the conſequents the diviſor.

Ex. How many rees are equal in value to a piaſtre of 272 mervadies?

Antecedents.		*Conſequents.*
94 piaſtres	=	100 ducats.
1 ducat	=	320 mervadies.
272 mervad.	=	how many rees?
400 rees	=	50 d. Flemiſh.
56 d. Flem.	=	1 crown.
3 crowns	=	94 d. Sterling.
37500 d. Ster.	=	680 piaſtres.

$$\frac{94 \times 272 \times 400 \times 56 \times 3 \times 37500}{100 \times 320 \times 50 \times 94 \times 680} = \frac{14 \times 3 \times 15}{1} = 630$$

rees. *Anſ.*

Note, The value of an antecedent is always of the ſame denomination with the preceding conſequent; and the value of a conſequent is of the ſame denomination with the ſubſequent antecedent.

In this circular exchange, the agents or factors at the different places through which the money paſſes, retain ſo much in name of commiſſion; and the regular accu-

 rate

rate method of computation is, to deduce the commiſsion from the ſeveral conſequents.

Thus, if we reſume the former queſtion, and ſuppoſe the commiſſion to be $\frac{1}{2}$ *per cent.* the antecedents and conſequents, with commiſſion deduced, will ſtand as under.

Antecedents.		*Conſequents.*
94 piaſtres	=	99.5 ducats, com. deduced.
1 ducat	=	318.4 mervadies, com. ded.
272 mervad.	=	626.85 rees, com. ded.
400 rees	=	49.75 d. Flem. com. ded.
56 d. Flem.	=	.995 crowns, com. ded.
3 crowns	=	94 d. Ster.
How many d. Ster.	=	1 piaſtre ?

By working as formerly, the arbitrated price, or value, of the piaſtre, will be found to be 53.78 d. Sterling nearly, which is ſomething leſs than the anſwer found, excluſive of charges. But ſtill there will be profit by the circular remittance : For

	L.	*s.*	*d.*
680 piaſtres at 53.78 d. Ster. =	152	7	6
———— at 50 d. Ster. =	141	13	4
Gain	10	14	2

The value of the above piaſtre, with commiſſion allowed, may be found eaſily, and pretty nearly, by deducing five commiſſions, equal to $2\frac{1}{2}$ *per cent.* from the arbitrated price, excluſive of charges, thus :

$$55\tfrac{5}{34} = 55.147$$
$$2\tfrac{1}{2}\ per\ cent. = \tfrac{1}{40} \mid 1.378$$
$$53.769 \text{ value nearly.}$$

Thus a perſon who knows at what rate he can draw or remit directly, and at the ſame time has advice of the courſe of exchange in foreign places, may chalk out a path for circulating his money, ſo as to make a benefit of

of his ſkill and credit: and herein lies the great art of ſuch negotiations.

Not only may different ſorts of money be equated in the manner above deſcribed, but alſo weights and meaſures.

EXAMPLE I.

If 102 lb. of Hamburgh be equal to 100 lb. of Amſterdam, and 100 of Amſterdam to 98 at Francfort, and 98 at Francfort to 105 at Leipſic, and 105 of Leipſic to 145 at Leghorn, and 145 at Leghorn to 106 at Cadiz, and 100 at Cadiz to $103\frac{1}{3}$ at London; how many lbs. at London are equal to 3060 at Hamburgh?

Antecedents.		*Conſequents.*
102 at Hamburgh	=	100 at Amſterdam
100 at Amſterdam	=	98 at Francfort.
98 at Francfort	=	105 at Leipſic.
105 at Leipſic	=	145 at Leghorn.
145 at Leghorn	=	106 at Cadiz.
300 at Cadiz	=	310 at London.
How many at London	=	3060 at Hamburgh?

$$\frac{100 \times 98 \times 105 \times 145 \times 106 \times 310 \times 3060}{102 \times 100 \times 98 \times 105 \times 145 \times 300} = \frac{106 \times 310 \times 3060}{102 \times 300} =$$

$\frac{55862}{17}$, and 17)55862(3286 lb. *Anſ.*

EXAMPLE II.

If $1\frac{1}{5}$ ells, or aunes, of Hamburgh, make 1 ell in Holland, and 7 in Holland make 4 in France, and 7 in France make 5 yards in England; how many yards in England are equal to 588 ells or aunes at Hamburgh? and what their price, at the rate of 4 l. Sterling for 5 yards Engliſh?

	Antecedents.		*Conſequents.*
$1\frac{1}{5} \times 5 =$	6 of Hamburgh	=	5 of Holland $= 1 \times 5$
	7 of Holland	=	4 of France.
	7 of France	=	5 of England.
	How many yards of England	=	588 of Hamburgh?

$\frac{5 \times 4 \times 5 \times 588}{6 \times 7 \times 7} = \frac{5 \times 4 \times 5 \times 2}{1} = 200$ yards. *Anſ.*

Then

Then for the price:

Antecedents.		*Consequents.*
6 of Hamburgh	=	5 of Holland.
7 of Holland	=	4 of France.
7 of France	=	5 of England.
5 of England	=	4 l. Sterling.
How much Sterling	=	588 of Hamburgh?

$$\frac{5\times4\times5\times4\times588}{6\times7\times7\times5}=\frac{4\times5\times4\times2}{1}=160 \text{ l. Ster. } \textit{Anſ.}$$

EXAMPLE III.

Antecedents.		*Conſequents.*
If 82 buſhels at London	=	27 muddles at Amſter.
27 muddles at Amſter.	=	19 ſetiers at Paris,
38 ſetiers at Paris	=	65 veertels at Antwerp,
65 veertels at Antwerp	=	104 hanegas at Cadiz,
52 hanegas at Cadiz	=	216 alquiers at Liſbon.
How many alquiers at Liſb.	=	410 buſhels at London?

$$\frac{27\times19\times65\times104\times216\times410}{82\times27\times38\times65\times52}=108\times10=1080$$ alquiers at Liſbon. *Anſ.*

Though foreign weights and meaſures may be equated with thoſe of Britain by arbitration, as in the above examples; yet this is done more eaſily and readily by the rule of three, from the following tables, which exhibit the proportion or conformity that the weights, long meaſures, and corn-meaſures, of the principal places in Europe, have with thoſe of London and Amſterdam.

TABLE

Table I. Of WEIGHTS.

100 lb. of England are equal to		100 lb. of Amsterdam are equal to		At
lb.	*oz.*	*lb.*	*oz.*	
100	0	109	8	London
91	8	100	0	Amsterdam
96	8	105	8	Antwerp
88	0	96	4	Rouen
106	0	116	0	Lyons
90	9	99	0	Rochelle
107	11	118	0	Toulouse
113	0	123	8	Marseilles
81	7	89	0	Geneva
93	5	102	0	Hamburgh.
89	7	98	0	Francfort
96	1	105	0	Leipsick
137	4	150	0	Genoa
132	11	145	0	Leghorn
153	11	168	0	Milan
152	0	166	0	Venice
154	10	169	0	Naples
97	0	106	0	Seville
97	0	106	0	Cadiz
104	13	114	8	Portugal
96	5	105	4	Liege
112	$0\frac{2}{3}$	123	$0\frac{1}{12}$	Russia
107	$0\frac{1}{24}$	117	0	Sweden
89	$0\frac{1}{2}$	97	13	Denmark

As the weights of Amsterdam, Paris, Bourdeaux, Strasburgh, Besançon, and several other places, have but a very minute difference, they are all comprehended under that of Amsterdam, as that of Nuremberg is under Francfort, and others in like manner.

TABLE

TABLE II. Of LONG MEASURE.

100 ells or aunes, or 125 yards of England, are equal to	100 aunes of Amsterdam are equal to	At
Aunes.	*Aunes.*	
100	60	London
166⅔	100	Amsterdam
162 4/9	98¾	Brabant
97 1/12	58½	France
200 1/24	120	Hamburgh
208⅓	125	Breslau in Silesia
187½	112½	Dantzick
183¼	110	Bergue and Drontheim
194½	116⅔	Sweden
143 1/7	86	St Gall for linen
186⅔	112	St Gall for cloth
100	60	Geneva
164½	98¾	Brussels, Antwerp
Canes.	*Canes.*	
58¼	35	Marseilles
62⅗	37½	Toulouse
50 13/15	30½	Genoa
55⅓	33	Rome
Varas.	*Varas.*	
133⅓	80	Spain
101⅔	61	Portugal
Cavidos.	*Cavidos.*	
166⅔	100	Portugal
Brasses.	*Brasses.*	
170⅓	102	Venice
174½	105¼	Bergamo
194 1/7	117	Florence, Leghorn, Lucca
214⅙	128¼	Milan
Arsheens.	*Arsheens.*	
178 4/7	297 15/21	Petersburgh

The

The aunes or ells of Haerlem, Leyden, the Hague, Rotterdam, and other places of Holland, alſo that of Nuremberg, are all equal to that of Amſterdam, and comprehended under it; the aune of Oſnaburg is equal to that of France; and the aunes of Bern, Baſil, Francfort, and Leipſic, are equal to that of Hamburgh.

Table III. Of CORN-MEASURES.

$10\frac{1}{4}$ quarters, or 82 Wincheſter buſhels of England, or 1 laſt of Amſterdam, make, at

Aiguillon, 41 ſacks
Albi, 25 ſetiers
Alicant, 12 cahizes
Alkmaar, 36 ſacks
Amersfort, 16 muddes.
Amſterdam, 27 muddles, or 1 laſt
Antwerp, $32\frac{1}{2}$ veertels
Arles, 49 ſetiers
Bayonne, 36 ſacks
Beaucaire, 28 ſetiers
Beaumont, 38 ſacks
Bergen-op-zoom, 63 ſiſters
Bois-le-Duc, $20\frac{1}{2}$ mouwers
Bommel, 18 muddes
Bourdeaux, 38 boiſſeux
Breda, 33 veertels
Bruges, $17\frac{1}{2}$ hoedts
Bruſſels, 25 ſacks
Bueren, 21 muddes
Cadillac, $33\frac{1}{3}$ ſacks
Cadiz, 52 hanegas
Cahors, 100 cartes
Campen, $24\frac{1}{2}$ muddes
Carcaſſone, 35 ſetiers
Clairac, $34\frac{1}{2}$ ſacks
Cleves, $16\frac{1}{4}$ mouwers
Condom, 41 ſacks
Coningſberg, 1 laſt
Copenhagen, 42 tuns
Dantzick, 1 laſt
Delft, 29 ſacks
Deventer, 36 muddes
Doeſburg, 22 mouwers
Dort, 24 ſacks
Dunkirk, 18 razieres
Edam, 27 muddes
Elbing, 1 laſt
Emden, $15\frac{1}{4}$ tuns
Erfelſteyn, 21 muddes
Francfort, 27 malders
Ghent, 56 halſters
Genoa, 25 mines
Gimond, 20 ſacks
Graveline, 22 razieres
Haerlem, 38 ſacks
Hamburgh, $\frac{12}{13}$ of a laſt
Heuſden, $17\frac{1}{4}$ muddes
Hoorn, 44 ſacks
Ireland, 38 buſhels
La Brille, 40 ſacks
La Reole, 30 ſacks
Lavour, 21 ſetiers
Leyden, 44 ſacks
Libourne, 35 ſacks
Liege, 96 ſetiers.

$10\frac{1}{4}$ quarters, or 82 Winchester bushels of England, or 1 last of Amsterdam, make, at

Lisle, 38 razieres	Ruremonde, 68 schepels
Lisbon, 216 alquiers	Riga, 46 loopens
Leghorn, 40 sacks	Rotterdam, 29 sacks
Louvain, 27 muddes	St Giles, 40 charges
Lubeck, 95 schepels	St Omer, $22\frac{1}{2}$ razieres
Lyons, $14\frac{1}{4}$ anees	St Valery, 19 setiers
Middlebourgh, $41\frac{1}{2}$ sacks	Saumur, 19 setiers
Montfort, 21 muddes	Steenbergen, 35 veertels
Muyden, 44 sacks	Stockholm, 23 tuns
Naerden, 44 sacks	Terveer, 39 sacks
Nerac, $33\frac{1}{3}$ sacks	Thiel, 21 muddes
Nieuport, $17\frac{1}{2}$ razieres	Toulouse, 26 setiers
Oudewater, 21 muddes	Tongres, 15 muddes
Paris, 19 setiers	Tonningen, 24 tuns
Porto Port, 180 alquiers	Venloo, $21\frac{3}{4}$ mouwers
Purmerent, 27 muddes	Vianen, 20 muddes
Rabastens, 17 setiers	Utrecht, 25 muddes
Rhenen, 20 muddes	Zurickzee, 40 sacks

The use of the Tables.

EXAMPLE I.

What are 800 lb. of Lyons equal to in England?

Say, from Table I. As 106 lb. of Lyons, : 100 lb. of England :: 800 of Lyons : $754\frac{1}{2}$ of England.

EXAMPLE II.

How many aunes of Amsterdam are equal to 500 aunes of Dantzick?

Say, from Table II. As $112\frac{1}{2}$ aunes of Dantzick : 100 aunes of Amsterdam :: 500 aunes of Dantzick : $444\frac{1}{4}$ aunes of Amsterdam.

EXAMPLE III.

How many setiers will 400 bushels of England make at Paris?

Say,

Say, from Table III. As 82 bushels of England : 19 fetiers of Paris : : 400 bushels of England : $92\frac{1}{2}$ fetiers of Paris.

CHAP. VIII.

ALLIGATION.

ALligation serves to solve questions that relate to the mixing of simples; and is either *medial* or *alternate*.

I. *Alligation Medial.*

Alligation medial, from the rates and quantities of the simples given, discovers the rate of the mixture.

RULE.

As the total quantity of the simples,
To their price or value;
So any quantity of the mixture,
To the rate.

EXAMPLES.

1. A grocer mixes 30 lb. of currants, at 4 d. per lb. with 10 lb. of other currants, at 6 d. per lb.: What is the value of 1 lb. of the mixture? *Ans.* $4\frac{1}{2}$ d.

lb.	*d.*		*d.*
30,	at 4	amounts to	120
10,	at 6	———	60
40			180

	lb.		*d.*		*lb.*		*d.*
If	40	:	180	: :	1	:	$4\frac{1}{2}$

2. A farmer mingleth several sorts of wheat as follows, viz. 20 bushels, at 5 s. per bushel; 36 bushels,

at 3 s.; and 40 bushels, at 2 s.: What is a bushel of the mixture worth? *Ans.* 3 s.

Bush.	*s.*		*s.*
20,	at 5,	comes to	100
36,	at 3	———	108
40,	at 2	———	80
—			—
96			288

Bush. *s.* *Bush.* *s.*
If 96 : 288 :: 1 : 3

3. A tobacconist mixeth 30 lb. of tobacco, at 9 d. per lb. with 60 lb. at 14 d. and $24\frac{3}{4}$ lb. at 2 s. per lb.: What is a pound of the mixture worth?

lb.	*Rates.*	*L.*
30	× .0375	= 1.125
60	× .058$\dot{3}$	= 3.5
24.75	× .1	= 2.475
114.75		7.1

lb. *L.* *lb.* *L.*
If 114.75 : 7.1 :: 1 : .0618

$$
\begin{array}{l}
\qquad\qquad\qquad\quad s.\quad d.\\
114.75)7.1000(.0618 = 1 \quad 2\tfrac{3}{4}\\
\qquad\quad 68850\\
\qquad\quad \overline{21500}\\
\qquad\quad 11475\\
\qquad\quad \overline{100250}\\
\qquad\quad 91800\\
\qquad\quad \overline{8450}
\end{array}
$$

Note 1. When the quantity of each simple is the same, the rate of the mixture is readily found by adding the rates of the simples, and dividing their sum by the number of simples, thus.

Suppose

Suppose a grocer mixes several sorts of sugar, and of each an equal quantity, viz. at 50 s. at 54 s. and at 60 s. per C. the rate of the mixture will be 54 s. 8 d. per C.; for,

$$50 + 54 + 60 = 164, \text{ and } 3)164(\overset{s.}{54}\ \overset{d.}{8}$$

Note 2. If it be required to increase or diminish the quantity of the mixture, say, As the sum of the given quantities of the simples, to the several quantities given; so the quantity of the mixture proposed, to the quantities of the simples sought. Thus,

If it be required to increase the quantity of the mixture in Ex. 2. to 120 bushels, say,

				Bush.		*s.*
96 : 20	::	120	:	25,	at	5
96 : 36	::	120	:	45,	at	3
96 : 40	::	120	:	50,	at	2
		Proof		120		

Note 3. If it be required to know how much of each simple is in an assigned portion of the mixture, say, As the quantity of the mixture, to the several quantities of the simples given; so the quantity of the assigned portion, to the quantities of the simples sought. Thus,

Suppose a grocer mixes 10 lb. of raisins, with 30 lb. of almonds, and 40 lb. of currants, and it be demanded, how many ounces of each sort are found in every pound, or in every sixteen ounces of the mixture, say,

				oz.	
80 : 10	::	16	:	2	raisins.
80 : 30	::	16	:	6	almonds.
80 : 40	::	16	:	8	currants.
		Proof		16	

Note 4. If the rates of two simples, with the total value and total quantity of the mixture be given, the

 quantity

quantity of each simple may be found as follows, viz. multiply the lesser rate into the total quantity, subtract the product from the total value, and the remainder will be equal to the product of the excess of the higher rate above the lower, multiplied into the quantity of the higher-priced simple; and, consequently, the said remainder, divided by the difference of the rates, will quot the said quantity. Thus,

Suppose a grocer has a mixture of 400 lb. weight, that cost him 7 l. 10 s. consisting of raisins, at 4 d. per lb. and almonds at 6 d. how many pounds of almonds were in the mixture?

	lb.	*Rates.*	
L. s. d.	400	6 d.	
7 10 = 1800	4	4 d.	
1600	1600 d.	2 d.	*L. s.*
2)200(100 lb. of almonds, at 6 d. is,			2 10
And 300 lb. of raisins, at 4 d. is,			5 0
Total 400		Proof	7 10

MORE EXAMPLES.

4. A merchant mingled three different sorts of sugar, viz. 3 C. at 2 l. 16 s. per C. and 3 C. at 3 l. 14 s. 8 d. with 6 C. at 1 l. 17 s. 4 d.: What is a C. of the mixture worth? *Ans.* 2 l. 11 s. 4 d.

5. A vintner mingles several sorts of wine, viz. 36 gallons at 8 s. per gallon, 2 gallons at 7 s. per gallon, 13 gallons at 1 s. per gallon, with 12 gallons of water: What is a gallon of the mixture worth? *Ans.* 5 s.

6. If, with 40 bushels of corn at 4 s. per bushel, there are mixed 10 bushels at 6 s. per bushel, 30 bushels at 5 s. per bushel, and 20 bushels at 3 s. per bushel, what will 10 bushels of the mixture be worth? *Ans.* 2 l. 3 s.

7. A goldsmith mixes 30 ounces of gold 22 caracts fine, 10 ounces 20 caracts fine, and 20 ounces 18 caracts fine: How many caracts fine is the mixture? *Ans.* $20\frac{1}{3}$; that is, of the 24 parts or caracts into which the ounce

ounce is ſuppoſed to be divided, $20\frac{1}{3}$ are pure gold, and the remaining $3\frac{2}{3}$ are alloy.

Medicines are ſometimes mixed or compounded by this rule; for underſtanding the manner whereof it will be neceſſary to obſerve,

That medicines, drugs, or ſimples, with reſpect to their qualities, are divided into five ſorts, viz. hot, cold, dry, moiſt, temperate; and all, except the laſt, admit of four degrees, repreſented by indices, as in the following table.

Ind.	1	2	3	4	5	6	7	8	9
Deg.	4	3	2	1	0	1	2	3	4
	of cold and moiſture.					of heat and dryneſs.			

The index 3 denotes cold or moiſt in the 2d degree, and 9 denotes hot or dry in the 4th degree.

EXAMPLE.

If I have four ſimples, A, B, C, D, and mix them as follows, viz. 4 oz. of A hot in the 4th degree, 1 oz. of B hot in the 2d, 1 oz. of C temperate, and 3 oz. of D cold in the 3d degree; what will be the quality of the mixture or compound? *Anſ.* hot in the 1ſt degree.

$$
\begin{array}{lccc}
A & 4 \times 9 & = & 36 \\
B & 1 \times 7 & = & 7 \\
C & 1 \times 5 & = & 5 \\
D & 3 \times 2 & = & 6 \\
\hline
 & 9 & & 9)54(6
\end{array}
$$
hot in the 1ſt degree.

II. *Alligation Alternate.*

Alligation alternate, being the converſe of alligation medial, from the rates of the ſimples, and rate of the mixture given, finds the quantities of the ſimples.

RULES

RULES.

I. Place the rate of the mixture on the left ſide of a brace, as the root; and on the right ſide of the brace ſet the rates of the ſeveral ſimples, under one another, as the branches.

II. Link or alligate the branches, ſo as one greater, and another leſs than the root, may be linked or yoked together.

III. Set the difference betwixt the root and the ſeveral branches, right againſt their reſpective yoke-fellows. Theſe alternate differences are the quantities required.

Note 1. If any branch happen to have two or more yoke-fellows, the difference betwixt the root and theſe yoke-fellows muſt be placed right againſt the ſaid branch, one after another, and added into one ſum.

Note 2 In ſome queſtions, the branches may be alligated more ways than one; and a queſtion will always admit of ſo many anſwers, as there are different ways of linking the branches.

Alligation alternate admits of three varieties, viz. 1. The queſtion may be unlimited, with reſpect both to the quantity of the ſimples, and that of the mixture. 2. The queſtion may be limited to a certain quantity of one or more of the ſimples. 3. The queſtion may be limited to a certain quantity of the mixture.

Variety I.

When the queſtion is unlimited, with reſpect both to the quantity of the ſimples, and that of the mixture. This is called *Alligation Simple.*

EXAMPLES.

1. A grocer would mix ſugars, at 5 d. 7 d. and 10 d. per lb. ſo as to ſell the mixture or compound at 8 d. per lb.: What quantity of each muſt he take?

$$8\left\{\begin{array}{l}5\\7\\10\end{array}\right.\begin{array}{ll}2 & \\ 2 & \\ 3, & 1\end{array}\left|\begin{array}{l}lb.\\2\\2\\4\end{array}\right.$$

Here the rate of the mixture 8 is placed on the left ſide of the brace, as the root; and on the right ſide of the ſame brace are ſet the rates of the ſeveral ſimples, viz. 5, 7, 10, under one another, as the branches; according to Rule I.

The branch 10 being greater than the root, is alligated or linked with 7 and 5, both theſe being leſs than the root; as directed in Rule II.

The difference between the root 8 and the branch 5, viz. 3, is ſet right againſt this branch's yoke-fellow 10. The difference between 8 and 7 is likewiſe ſet right againſt the yoke-fellow 10. And the difference betwixt 8 and 10, viz. 2, is ſet right againſt the two yoke-fellows 7 and 5; as preſcribed by Rule III.

As the branch 10 has two differences on the right, viz. 3 and 1, they are added; and the anſwer to the queſtion is, that 2 lb. at 5 d. 2 lb. at 7 d. and 4 lb. at 10 d. will make the mixture required.

The truth and reaſon of the rules will appear by conſidering, that whatever is loſt upon any one branch is gained upon its yoke-fellow. Thus, in the above example, by ſelling 4 lb. of 10 d. ſugar at 8 d. per lb. there is 8 d. loſt: but the like ſum is gained upon its two yoke-fellows; for by ſelling 2 lb. of 5 d. ſugar at 8 d. per lb. there is 6 d. gained; and by ſelling 2 lb. of 7 d. ſugar at 8 d. there is 2 d. gained; and 6 d. and 2 d. make 8 d.

Hence it follows, that the rate of the mixture muſt always be mean or middle with reſpect to the rates of the ſimples; that is, it muſt be leſs than the greateſt, and greater than the leaſt; otherwiſe a ſolution would be impoſſible. And the price of the total quantity mixed, computed at the rate of the mixture, will always be equal to the ſum of the prices of the ſeveral quantities

caſt

caſt up at the reſpective rates of the ſimples. This may be uſed by way of proof; and the above example proved in this manner follows.

$$8\left\{\begin{array}{l}5\\7\\10\end{array}\right.\begin{array}{l}2\\2\\3,\ 1\end{array}\left|\begin{array}{l}2\times\ \ 5=10\\2\times\ \ 7=14\\4\times10=40\end{array}\right.$$

$$\overline{8}\times 8=64 \qquad \overline{64}\ \text{proof.}$$

Or ſay,

If 8 : 64 : : 1 : 8

That is, the ſum of the prices or price of the mixture, divided by the ſum of the quantities, quotes the rate of the mixture.

From the above proof it is obvious, that the rates of the ſimples, their total quantity, and the price of the mixture, being given; the rate of the mixture, and conſequently the quantities of the ſeveral ſimples may be found: for the price of the mixture divided by the total quantity of the ſimples, quotes the rate of the mixture; the alternate differences of which, and the ſeveral rates, are the quantities ſought.

The anſwer in the above example comes out to be 2, 2, 4; and as the branches can be linked but one way, we can, by the operation, have only this ſingle anſwer: but then any other three numbers that have the ſame proportion to one another, found by diviſion or multiplication, will alſo anſwer the queſtion. By this means the anſwer found may be increaſed or diminiſhed, till it ſuit the ſtock of ſimples on hand for making the mixture; as in the following ſpecimen.

$$\left.\begin{array}{l}1.\ 2.\ 3.\ 4.\ \ 5.\ \ 6.\ \ 7.\ \ 8.\ \ 9.\ 10.\ 11.\ 12.\ 13.\\1.\ 2.\ 3.\ 4.\ \ 5.\ \ 6.\ \ 7.\ \ 8.\ \ 9.\ 10.\ 11.\ 12.\ 13.\\2.\ 4.\ 6.\ 8.\ 10.\ 12.\ 14.\ 16.\ 18.\ 20.\ 22.\ 24.\ 26.\end{array}\right\}\ \&c.$$

But we may alſo find innumerable other anſwers, by increaſing or diminiſhing the alternate differences of any pair

pair of ſimples, without varying the reſt, by one or other of the two rules following.

I. When the rates of the variable pair of ſimples are both greater, or both leſs, than the rate of the mixture; increaſe the alternate difference of the one, by adding to it the difference betwixt the rate of the mixture and the rate of the other variable ſimple; and diminiſh the alternate difference of the other, by ſubtracting from it the difference betwixt the rate of the mixture and the rate of the former variable ſimple.

The alternate difference to be increaſed is optional, or which of them you pleaſe; and inſtead of the alternate differences, you may uſe any other numbers that are in the ſame proportion; and the higher theſe numbers are, the more anſwers they will afford. We ſhall exemplify the rule by the proportional numbers, 13, 13, 26, found above, making 13, 13, the variable pair, their rates, 5 and 7, being both leſs than 8, the rate of the mixture, and ſhall continue 26 unvaried.

By increaſing 13 in the upper line, we have,

13.	14.	15.	16.	17.
13.	10.	7.	4.	1.
26.	26.	26.	26.	26.

By increaſing 13 in the ſecond line, we have,

13.	12.	11.	10.	9.	8.	7.	6.	5.	4.	3.	2.	1.
13.	16.	19.	22.	25.	28.	31.	34.	37.	40.	43.	46.	49.
26.	26.	26.	26.	26.	26.	26.	26.	26.	26.	26.	26.	26.

II. When the rate of one of the variable ſimples is greater than the rate of the mixture, and the rate of the other leſs, their alternate differences, or rather the numbers proportional to them, muſt, as directed in the former rule, be both increaſed, or both diminiſhed, and either or each of theſe ways may be uſed.

We ſhall exemplify the rule, as formerly, by the proportional numbers 13, 13, 26, making the firſt 13 and 26 the variable pair, whoſe rates are 5 and 10, the

one below and the other above 8, the rate of the mixture, and ſhall continue the other 13 unvaried.

By increaſing both, we have,

13.	15.	17.	19.	21.	23.	25.	27.	29.	
13.	13.	13.	13.	13.	13.	13.	13.	13.	to infinity.
26.	29.	32.	35.	38.	41.	44.	47.	50.	

By diminiſhing both, we have,

13.	11.	9.	7.	5.	3.	1.
13.	13.	13.	13.	13.	13.	13.
26.	23.	20.	17.	14.	11.	8.

Again, we may make the ſecond 13 and 26 the variable pair, their rates being 7 and 10, the one below and the other above 8, the rate of the mixture, and continue the firſt 13 unvaried, as under.

By increaſing both, we have,

13.	13.	13.	13.	13.	13.	13.	13.	13.	
13.	15.	17.	19.	21.	23.	25.	27.	29	to infinity.
26.	27.	28.	29.	30.	31.	32.	33.	34.	

By diminiſhing both, we have,

13.	13.	13.	13.	13.	13.	13.
13.	11.	9.	7.	5.	3.	1.
26.	25.	24.	23.	22.	21.	20.

The reaſon of the rules is obvious; for by increaſing or diminiſhing the alternate differences, or other numbers proportional to them, in the manner preſcribed, the value of every pound of increaſe or diminution is the rate of the mixture.

2. A vintner would mix wines at 14 s. 15 s. 19 s. and 22 s. per gallon, ſo as the mixture may be worth 18 s. per gallon: What quantity of each muſt he take?

Firſt,

First, thus. *Gal.*

		Gal.
18	14	4
	15	1
	19	3
	22	4

Sec. thus. *Gal.*

		Gal.
18	14	1
	15	4
	19	4
	22	3

Thirdly, thus. *Gal.*

			Gal.
18	14	1,4	5
	15	1	1
	19	4,3	7
	22	4	4

Fourthly, thus. *Gal.*

			Gal.
18	14	4	4
	15	1,4	5
	19	3	3
	22	4,3	7

Fifthly, thus. *Gal.*

			Gal.
18	14	1,4	5
	15	4	4
	19	4	4
	22	4,3	7

Sixthly, thus. *Gal.*

			Gal.
18	14	1	1
	15	1,4	5
	19	4,3	7
	22	3	3

Seventhly, thus. *Gal.*

			Gal.
18	14	1,4	5
	15	1,4	5
	19	4,3	7
	22	4,3	7

Here the ſimples are linked ſeven different ways; and accordingly we have ſeven different anſwers to the queſtion; and, by multiplication or diviſion, each of theſe will produce other anſwers to infinity. Again, by taking each pair of ſimples ſucceſſively, and increaſing or diminiſhing their alternate differences, or other numbers proportional to them, in the manner taught above, we ſhall have other anſwers innumerable.

3. A grocer mixes teas, at 7 s. at 8 s. at 10 s. and at 13 s. per lb.: How much of each ſort muſt he take, that a pound of the mixture may be worth 9 s. 6 d.?

First, *lb.*

		lb.
9·5	7	3·5
	8	·5
	10	1·5
	13	2·5

Secondly, *lb.*

		lb.
9·5	7	·5
	8	3·5
	10	2·5
	13	1·5

Thirdly,

			lb.
9·5	7	·5,3·5	4
	8	·5	·5
	10	2·5,1·5	4
	13	2·5	2·5

Fourthly,

			lb.
9·5	7	3·5	3·5
	8	·5,3·5	4
	10	1·5	1·5
	13	2·5,1·5	4

Fifthly,

			lb.
9·5	7	·5,2·5	4
	8	3·5	3·5
	10	2·5	2·5
	13	2·5,1·5	4

Sixthly,

			lb.
9·5	7	·5	·5
	8	·5,3·5	4
	10	2·5,1·5	4
	13	1·5	1·5

Seventhly,

			lb.
9·5	7	·5,3·5	4
	8	·5,3·5	4
	10	2·5,1·5	4
	13	2·5,1·5	4

Other answers innumerable may be found by the methods explained above.

4. An apothecary would mix four ſimples, viz. A hot in the fourth degree, B hot in the ſecond, C temperate, and D cold in the third, ſo as the mixture may be hot in the firſt degree: What quantity of each ſimple muſt he take? See the table on compoſition of medicines in alligation medial.

6	9	1	A	1×9 = 9
	7	4	B	4×7 = 28
	5	3	C	3×5 = 15
	2	1	D	1×2 = 2

6×9 = 54 Proof 54

The branches may be alligated other ſix ways, which will afford the like number of other anſwers.

Variety II.

When the queſtion is limited to a certain quantity of one or more of the ſimples. This is called *Alligation Partial.*

If the quantity of one of the ſimples only be limited, alligate the branches, and take their differences, as if there

there had been no ſuch limitation; and then work by the following proportion.

As the difference right againſt the rate of the ſimple whoſe quantity is given,
To the other differences reſpectively;
So the quantity given,
To the ſeveral quantities ſought.

EXAMPLES.

1. A diſtiller would, with 40 gallons of brandy, at 12 s. per gallon, mix rum at 7 s. per gallon, and gin at 4 s. per gallon: How much of the rum and gin muſt he take, to ſell the mixture at 8 s. per gallon?

					Gal.	
8	12	1,4	5	40 of brandy.	Anſ.
	7	4	4	32 of rum.	
	4	4	4	32 of gin.	

The operation gives for anſwer 5 gallons of brandy, 4 of rum, and 4 of gin. But the queſtion limits the quantity of brandy to 40 gallons; therefore ſay,

If 5 : 4 :: 40 : 32

The quantity of gin, by the operation, being alſo 4, the proportion needs not be repeated. The proof is the ſame as formerly.

As the branches can be linked only one way, we can have but one anſwer by the operation; and becauſe the quantity of brandy is limited to 40 gallons, there can be no recourſe to proportional numbers; but by proceeding with the remaining pair of ſimples in the manner formerly taught, we may have a great many other anſwers, as follows, viz.

By increaſing the firſt 32, we have,

40.	40.	40.	40.	40.	40.	40.	40.	&c.
32.	36.	40.	44.	48.	52.	56.	60.	
32.	31.	30.	29.	28.	27.	26.	25.	

By

By increaſing the laſt 32, we have,

40. 40. 40. 40. 40. 40. 40. 40.
32. 28. 24. 20. 16. 12. 8. 4.
32. 33. 34. 35. 36. 37. 38 39.

2. An innkeeper would, with 72 buſhels of his beſt corn, at 60 d. per buſhel, mix other corns at 48 d. at 36 d. at 24 d. per buſhel: What quantity of the laſt three muſt he take, that the mixture may be worth 42 d. per buſhel ?

Firſt.

			Anſ.
42	60	36	72
	48	6	12
	36	6	12
	24	18	36

Secondly,

			Anſ.
42	60	6	72
	48	18	216
	36	18	216
	24	6	72

Thirdly,

				Anſ.
42	60	6,18	24	72
	48	6	6	18
	36	18,6	24	72
	24	18	18	54

Fourthly.

				Anſ.
42	60	18	18	72
	48	6,18	24	96
	36	6	6	24
	24	18,6	24	96

Fifthly,

				Anſ.
42	60	6,18	24	72
	48	18	18	54
	36	8	18	54
	24	18,6	24	72

Sixthly,

				Anſ.
42	60	6	6	72
	48	6,18	24	288
	36	18, 6	24	288
	24	6	6	72

Seventhly,

				Anſ.
42	60	6,18	24	72
	48	6,18	24	72
	36	18, 6	24	72
	24	18, 6	24	72

In each of the ſeven anſwers here given, 72 is limited ; but then any two of the three remaining ſimples may be eſteemed a pair, affording in all three pairs, from which innumerable other anſwers may be found.

3.

3. A goldſmith would, with 85 oz. of gold, worth 5 l. per oz. mix other ſorts of gold at 4 l. 10 s. per oz. and at 4 l. per oz.: What quantity of the laſt two, and what quantity of alloy muſt he take, that the maſs may be worth 4 l. 5 s. per oz.?

$$
\begin{array}{c}
 & \text{L.} & \\
4.25 & \left\{\begin{array}{l} 5 \\ 4.5 \\ 4 \\ 0 \end{array}\right. & \begin{array}{l} 4.25 \\ .25 \\ .25 \\ .75 \end{array}
\end{array}
$$

$$
\text{As } 4.25 : 85 :: \left\{\begin{array}{rcl} 4.25 & : & 85 \\ .25 & : & 5 \\ .25 & : & 5 \\ .75 & : & 15 \end{array}\right\} \textit{Anſ.}
$$

(oz. above the second column)

By other ways of linking, and from the pairs that ariſe from the three variable ſimples, other anſwers innumerable may be found.

4. A vintner would, with 10 gallons of wine at 12 s. per gallon, and 20 gallons at 15 s. per gallon, mix other wines, at 18 s. and at 20 s. per gallon: What quantity of the laſt two muſt he take, that the mixture may be worth 16 s. per gallon?

This queſtion is limited to a certain quantity of two of the ſimples, and, previous to the operation, the two given quantities muſt be ſuppoſed to be mixed by themſelves; and the rate of the mixture found by alligation medial as under.

Gallons.		s.
10,	at 12 s. come to	120
20,	at 15 s. amount to	300
30		30)420(14 s. per gallon.

The queſtion now may be conſidered as limited only in the quantity of one of the ſimples, and may be propoſed and ſolved as follows.

A

A vintner would, with 30 gallons of wine at 14 s. per gallon, mix other wines at 18 s. and at 20 s. per gallon: What quantity of the laſt two muſt he take, that the mixture may be worth 16 s. per gallon?

$$
\begin{array}{r|l|l|c|c}
 & & & & \textit{Anſ.} \\
 & 14 & 2,4 & 6 & 30 \\
16 & 18 & 2 & 2 & 10 \\
 & 20 & 2 & 2 & 10
\end{array}
$$

That is, 30 gallons of wine at 14 s. per gallon; or, reſolving this compound into its ſimples, the anſwer is, that 10 gallons of wine at 12 s. per gallon, and 20 gallons at 15 s. together with 10 gallons of each of the other two ſorts of wine, will make the mixture propoſed.

$$
\begin{array}{rcrcr}
10 & \times & 12 & = & 120 \\
20 & \times & 15 & = & 300 \\
10 & \times & 18 & = & 180 \\
10 & \times & 20 & = & 200 \\
\hline
50 & & & & 50)800(16 \text{ proof.}
\end{array}
$$

The quantity 30, or rather 10 and 20, are limited; but from the remaining pair a great many other anſwers may be found.

The procedure is the ſame when the queſtion is limited in the quantity of three or more of the ſimples.

Variety III.

When the queſtion is limited to a certain quantity of the mixture. This is called *Alligation Total.*

After linking the branches, and taking the differences, work by the proportion following.

As the ſum of the differences,
To each particular difference;
So the given total of the mixture,
To the reſpective quantities required.

EX-

EXAMPLES.

1. A vintner hath wine at 3 s. per gallon, and would mix it with water, ſo as to make a compoſition of 144 gallons, worth 2 s. 6 d. per gallon: How much wine and how much water muſt he take?

$$30\left\{\begin{array}{l}36\\0\end{array}\right)\begin{array}{r}30\\6\\\hline 36\end{array}\left|\begin{array}{rl}&\textit{Gal.}\\120&\text{of wine.}\\24&\text{of water.}\\\hline 144&\text{total.}\end{array}\right.\Bigg\}\,\textit{Anſ.}$$

$$\begin{array}{rl}120\times36=&4320\\24\times\ \ 0=&0\\\hline\text{Proof }144)&4320(30\end{array}$$

As 36 : 30 :: 144 : 120
As 36 : 6 :: 144 : 24

There being here only two ſimples, and the total of the mixture limited, the queſtion admits but of one anſwer.

2. A diſtiller would mix brandy at 8 s. wine at 7 s. and cyder at 1 s. per gallon, ſo that the mixture may contain 26 gallons, and be worth 5 s. per gallon: What quantity of each muſt he take?

$$5\left\{\begin{array}{l}8\\7\\1\end{array}\right)\begin{array}{l}4\\4\\3,2\end{array}\left|\begin{array}{r}4\\4\\5\\\hline 13\end{array}\right|\begin{array}{r}\textit{Anſ.}\\8\\8\\10\\\hline 26\end{array}\Bigg|$$

If 13 : 4 :: 26 : 8
If 13 : 5 :: 26 : 10

From the alternate differences, an immenſe variety of other unlimited anſwers may be obtained by the methods explained above; and any one of theſe unlimited anſwers, that ſuits the purpoſe beſt, may be reduced to the limits of the queſtion by the proportion aſſigned above. And even from the limited anſwer, other anſwers may be found, by obſerving the directions following, viz.

When three ſimples, as here, are mixed, though the total be limited, yet from the anſwer found by the operation, other anſwers may be deduced in the manner following.

Firſt, Increaſe or diminiſh the quantity of that ſimple whoſe rate alone is greater or leſs than the rate of the mixture, by the difference of the differences between the rate of the mixture and the rates of the other two ſimples. Thus, in the above example, the rate of the cyder alone is leſs than the rate of the mixture, and the differences betwixt the rate of the mixture and the rates of the other two ſimples, are reſpectively 3 and 2, whoſe difference is 1; and accordingly the quantity of the cyder found by the operation, viz. 10, may be increaſed or diminiſhed by 1, that is, it may be made 11 or 9.

Secondly, Of the two remaining ſimples, increaſe or diminiſh the quantity of that one whoſe rate is fartheſt from the rate of the mixture, by the ſum of the differences betwixt the rate of the mixture and the rates of the other two ſimples. Thus, in the above example of the two remaining ſimples, the rate of the brandy is fartheſt from the rate of the mixture, and the rate of the mixture differs from the rates of the other two ſimples, viz. the wine and cyder reſpectively, by 2 and 4, whoſe ſum is 6; and accordingly the quantity of the brandy, viz. 8, may be increaſed or diminiſhed by 6, that is, it may be made 14 or 2.

Thirdly, Diminiſh or increaſe the quantity of the remaining ſimple, by the ſum of the differences betwixt the rate of the mixture and the rates of the other two ſimples. But here obſerve, that the quantity of this ſimple is to be diminiſhed when thoſe of the former two are increaſed; and the contrary. Thus the quantity of the wine, viz. 8, is to be diminiſhed or increaſed by 7, the ſum of 3 and 4, the reſpective differences betwixt the rate of the mixture and the rates of the brandy and cyder; that is, it may be made 1 or 15.

By this means other two anſwers to the above queſtion are obtained, which, with the anſwer found by the operation, are here ſubjoined.

Brandy

Brandy	14.	8.	2.
Wine	1.	8.	15.
Cyder	11.	10.	9.
	26	26	26

3. A goldſmith hath ſeveral ſorts of gold, viz. N° 1. of 24 caracts fine, N° 2. of 22 caracts fine, N° 3. of 18 caracts fine, and N° 4 of 16 caracts fine: How much of each ſort muſt he take, to take a maſs of 60 ounces, of 21 caracts fine?

			Anſ.	
21	24	5	25	As 12 : 5 :: 60 : 25
	22	3	15	As 12 : 3 :: 60 : 15
	18	1	5	As 12 : 1 :: 60 : 5
	16	3	15	
		12	60	

The branches in the above example may be linked other ſix ways, which will accordingly afford ſo many other anſwers.

When four ſimples, as here, are mixed, and it is required to deduce other anſwers from that found by the operation, in this caſe one of the ſimples muſt be conſidered as invariable, and the three variable ſimples muſt be ſuch, that the rate of one of them may, with reſpect to the rate of the mixture, be oppoſite to the rates of the other two; that is, if the rate of one of the variable ſimples be greater than the rate of the mixture, then the rates of the other two muſt both be leſs; and the contrary.

Thus, in the above example, making N° 1. invariable, the rate of N° 2. is greater than the rate of the mixture, and the rates of the other two are both leſs; and accordingly, by the three directions ſubjoined to Ex. 2. we may, from the anſwer in Ex. 3. deduce other anſwers, as follows.

N° 1.	25.	25.	25.	25.
N° 2.	15.	13.	11.	9.
N° 3.	5.	11.	17.	23.
N° 4.	15.	11.	7.	3.
	60	60	60	60

Next, making N° 2. invariable, we ſhall have,

N° 1.	25.	23.	21.
N° 2.	15.	15.	15.
N° 3.	5.	13.	21.
N° 4.	15.	9.	3.
	60	60	60

Again, making N° 3. invariable, we ſhall have,

N° 1.	31.	25.	19.	13.	7.	1.
N° 2.	7.	15.	23.	31.	39.	47.
N° 3.	5.	5.	5.	5.	5.	5.
N° 4.	17.	15.	13.	11.	9.	7.
	60	60	60	60	60	60

Laſtly, making N° 4. invariable, we ſhall have,

N° 1.	17.	21.	25.	29.	33.
N° 2.	27.	21.	15.	9.	3.
N° 3.	1.	3.	5.	7.	9.
N° 4.	15.	15.	15.	15.	15.
	60	60	60	60	60

By taking the ſix anſwers that will ariſe from the other ſix ways of linking, and proceeding in like manner with each of them, a great many other anſwers may be found.

4. A merchant having wines at 12 d. 14 d. 15 d. 18 d. and 22 d. per pint, would make a mixture of 1240 pints, to ſell at 17 d. per pint: What quantity of each muſt he take?

				Anf.
17	12	1,5	6	240
	14	5	5	200
	15	5	5	200
	18	5	5	200
	22	5,3,2	10	400
			31	1240

As 31 : 6 :: 1240 : 240
As 31 : 5 :: 1240 : 200
As 31 : 10 :: 1240 : 400

The branches in this example may be alligated a great many other ways; each of which will afford a different anfwer.

When five fimples, as here, are mixed; in order to deduce other anfwers from that found by the operation, two of the fimples, taken by turns, muft always be made invariable; and the three remaining fimples muft be fuch, that the rate of one of them being greater or lefs than the rate of the mixture, the rates of the other two muft refpectively be both lefs, or both greater, than the rate of the mixture; and then the procedure is the fame as formerly. In like manner, when fix, feven, eight, &c. fimples, are mixed, all the fimples muft by turns be made invariable except three; and thefe three muft be qualified as above.

5. Suppofe 9 lb. of pure gold immerfed in a veffel full of water to expel 3 lb. of water, 9 lb. of pure filver to expel 6 lb. of water, and 9 lb. of a mafs made up of gold and filver to expel 4 lb. of water; required the quantity of gold and filver in the faid mafs.

In order to folve this queftion, the three feveral quantities of water expelled, which denote the fpecific bulks or gravities of the fimples, muft be confidered as the rates; that of the mafs being efteemed the root, and the other two the branches, as follows.

Ans.

$$4\left\{\begin{array}{c}3\\6\end{array}\right)\begin{array}{c}2\\1\\\hline 3\end{array}\left|\begin{array}{l}6\text{ lb. of pure gold}\\3\text{ lb. of pure silver}\\\hline 9\end{array}\right\}\text{ in the mass.}$$

Hence is solved the famous question with respect to the crown of Hiero King of Syracuse. He had ordered a crown to be made of pure gold; but suspecting that the founder had mixed silver with the gold, he desired Archimedes to examine the affair, and tell him, how much gold and how much silver was in the crown. Archimedes, by immersing the crown, and also the like quantities of pure gold and silver, in water, severally, and observing the respective quantities of water expelled, found an answer to the question.

CHAP. IX.

RULE of FALSE.

THE Rule of False is so called, because, by the help of false supposed numbers, the true ones are discovered; and is divided into Single and Double, or into Single Position and Double Position.

Before the knowledge of algebra came to be common, this rule was in high esteem, and found to be of great service; and though not so universally practised now as formerly, yet still it continues to be of considerable use.

I. *Single Position.*

Single position is, when, by one supposition, or one position, the number sought is discovered: in the finding of which we work first with the position, as if it was the true number; and, having brought out the result, we then proceed by the following proportion.

As

As the refult by operation, or falfe refult,
To the falfe pofition ;
So the true refult,
To the true pofition, or number fought.

EXAMPLES.

1. A, B, and C, buy a quantity of wine for 340 l.; of which A pays three times more than B, and B four times more than C: What paid each?

	L.			
Suppofe A paid	36	As 51 : 36 : : 340 : 240		
Then B paid	12			*L.*
And C paid	3		So A paid	240
Refult	51		B paid	80
			C paid	20
				340 proof.

2. A perfon having about him a certain number of crowns, faid, if a third, a fourth, and a fixth of them were added together, the fum would be 45: How many crowns had he?

Suppofe 48, whereof a third is	16
a fourth is	12
a fixth is	8
Refult	36

As 36 : 48 : : 45 : 60	A third of 60 is	20
	a fourth is	15
	a fixth is	10
	Proof	45

3. Suppofe the fum of 20 l. was to be paid by A, B, and C, in the following proportion, viz. A $\frac{1}{2}$, B $\frac{1}{3}$, C $\frac{1}{4}$: What muft each pay?

L.

	L.	s.	d.
$\frac{1}{2}$ of 20 l. is	10	0	0
$\frac{1}{3}$ is	6	13	4
$\frac{1}{4}$ is	5	0	0
Result	21	13	4

$$\text{As } \begin{matrix} L. & s. & d. \\ 21 & 13 & 4 \end{matrix} : \begin{Bmatrix} L. & s. & d. \\ 10 & 0 & 0 \\ 6 & 13 & 4 \\ 5 & 0 & 0 \end{Bmatrix} :: \begin{matrix} L. \\ 20 \end{matrix} : \begin{Bmatrix} L. & s. & d. \\ 9 & 4 & 7\frac{5}{13} \\ 6 & 3 & 0\frac{12}{13} \\ 4 & 12 & 3\frac{9}{13} \end{Bmatrix}$$

Proof 20 0 0

4. A schoolmaster being asked how many scholars he had, answered, If I had as many, half as many, and one fourth as many, I should have 99: How many had he? *Ans.* 36.

5. A, B, and C, discoursing of their age, B says to A, I am as old, and half as old again, as you; says C to B, I am twice as old as you; and A affirms, that the sum of their three ages amounted to 165: What was each man's age? *Ans.* A 30, B 45, and C 90 years of age.

6. A gentleman bought a chaise, horse, and harness, for 55 l.; the horse came to twice the price of the chaise, and the harness cost one third the price of the horse: What did he pay for each? *Ans.* For the chaise 15 l. for the horse 30 l. and for the harness 10 l.

II. *Double Position.*

Double position is when two suppositions, or two positions, must be used, in order to discover the number sought: in finding whereof, work with each position, as if it was the true number, till by this means you bring out the results, and consequently the errors; and then proceed by the following

RULE.

Multiply the two positions into their altern errors; then,

then, errors alike, that is, both exceſſes, or both defects, divide the difference of the products by the difference of the errors. But if the errors are unlike, that is, the one an exceſs and the other a defect, divide the ſum of the products by the ſum of the errors.

Or, inſtead of the above rule, you may uſe the following proportion.

As the difference of the errors, if alike, or their ſum, if unlike,
To the difference of the poſitions;
So either error,
To a fourth number.

Which fourth number added to, or ſubſtracted from, the poſition producing the error, gives the number ſought.

EXAMPLES.

1. A perſon buys 30 pints of liquor for 50 ſhillings; part beer, at 6 d. per pint, and part wine, at 3 s. per pint: How much was there of each?

	Pts.		s.	
Poſition 1. Beer	6,	at 6 d. is	3	
Then wine	24,	at 3 s. is	72	
			75	reſult.
			50	
			+ 25	error of exceſs.

	Pts.		s.	
Poſition 2. Beer	10,	at 6 d. is	5	
Then wine	20,	at 3 s. is	60	
			65	reſult.
			50	
			+ 15	error of exceſs.

Poſ. Er.

$6 \times 15 = 90$

$10 \times 25 = 250$

Pts.

10) 160(16 beer.

	Pts.		s.
Beer	16,	at 6 d. is	8
Wine	14,	at 3 s. is	42
		Proof	50

 Or,

Or, work by the proportion, as follows.

As 10 : 4 :: 25 : 10
Or, As 10 : 4 :: 15 : 6

And $10 + 6 = 16$ pints of beer.
And $6 + 10 = 16$ pints of beer, as before.

It is eaſy to perceive, that the fourth number muſt here be added to the poſition, becauſe, by increaſing the quantity of beer, the quantity of wine is leſſened; and this of courſe leſſens the reſult, which is too great.

The poſitions might have been ſo choſen, that the errors thence ariſing would have been both defects, or the one an exceſs, and the other a defect; but the doing of this is left as a proper exerciſe to the learner. Theſe varieties ſhall be illuſtrated in other examples.

It is worth while here alſo to obſerve, that double poſition takes place, or becomes neceſſary, when there is no partition of prices or quantities given, whereby to found or apply a proportion; for the queſtion ſays, A perſon bought 30 pints of liquor, part beer, part wine, without partitioning the quantities; whereas, had the queſtion ſaid, That for every 8 pints of beer, he had 7 pints of wine, the anſwer might have been found by ſingle poſition.

The reaſon of the aſſigned proportion will appear by conſidering, that the reſult varies with the poſition; and that the error of a reſult is ever proportional to the error of its poſition; ſo that the error of any one reſult is to the error of its own poſition, as the error of any other reſult is to the error of its poſition; and the difference of any two reſults has the ſame proportion to the difference of the poſitions from which they flow, as the difference of any other two reſults has to the difference of their poſitions. But the difference of the errors is equal to the difference of the reſults; therefore, As the difference of the errors, if alike, to the difference of the poſitions, ſo either error to a fourth number, viz. the error of the poſition.

When

When the errors are unlike, one of the falfe refults is greater and the other lefs than the true refult given in the queftion; and confequently the difference of the errors is found the fame way as the difference of an affirmative and negative quantity, namely, by taking their fum; or, in the fame manner as is found the difference of latitude of two places, whereof one lies to the north and the other to the fouth of the equator; that is, by adding the latitudes of the two places into one fum.

The truth of the rule will appear, by obferving, that the difference of the two products arifing from the multiplication of the pofitions into their altern errors, may be conceived as made up of two fmall parts or products viz. firft, the product of halt the fum of the pofitions into the difference of the errors; fecondly, the product of half the fum of the errors into the difference of the pofitions. Now, the former of thefe fmall products, divided by the difference of the errors, quotes half the fum of the pofitions, the divifor being one of the factors; the latter of thefe fmall products, divided alfo by the difference of the errors, quotes half the fum of the errors of the pofitions, by the proportion; and confequently the fum of thefe two fmall products, (equal to the difference of the alternate products), divided by the difference of the errors, quotes the true pofition, or anfwer fought, when the errors are alike.

When the errors are unlike, their difference, as alfo the difference of the alternate products, is found by taking their fum, as has been already fhown.

2. A, B, and C, built a houfe, which coft 76 l.; whereof A paid a fum unknown, B paid 10 l. more than A, and C paid as much as A and B: What did each partner pay?

		L.			L.
Position 1.	A	6	*Position* 2.	A	9
	B	16		B	19
	C	22		C	28
Result		44	Result		56
		76			76
Error of defect		— 32	Error of defect		— 29

Pos. *Er.*

6 × 20 = 120

9 × 32 = 288

L.

12)168(14 A

	L.
A	14
B	24
C	38
	76 proof.

By the proportion,

As 12 : 3 :: 32 : 8. And 8 + 6 = 14 } A's share,
Or, As 12 : 3 :: 20 : 5. And 5 + 9 = 14 } as before.

It is obvious, that the fourth numbers here must be added to the positions, because the results were both too small.

3. A labourer having threshed out 40 quarters of grain, part wheat, part barley, received, in name of wages, 28 s. or 336 d. being paid at the rate of 12 d. for every quarter of wheat, and 6 d. for every quarter of barley: How many of the 40 quarters were wheat, and how many were barley?

		Q.		d.
Position 1.	Wheat	10,	at 12 d. is,	120
	Barley	30,	at 6 d. is,	180
			Result	300
				336
			Error of defect	— 36

Position

		Q.				d.
Position 2.	Wheat	26,	at	12 d.	is	312
	Barley	14,	at	6 d.	is	84
					Result	396
						336
					Error of excess +	60

Pos. *Er.*

$10 \times 60 = 600$

$26 \times 36 = 936$

$96)\ 1536(16$ quarters of wheat.

96

576

576

(0)

	Q.					d.
Wheat	16,	at	12 d.	is		192
Barley	24,	at	6 d.	is		144
					Proof	336

By the proportion,

As $96 : 16 :: 36 : 6$

Or, As $96 : 16 :: 60 : 10$

And $6 + 10 = 16$ } quarters of wheat,
And $26 - 10 = 16$ } as before.

The fourth number of the first proportion is added to the position, because the result was too little. But the fourth number of the second proportion is subtracted from the position, the result being too great.

4. What number is that whereof $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$, make 19?

1 pos.

1 pos. 30.	2 pos. 10.
$\frac{1}{3} = 10$	$\frac{1}{3} = 3.\dot{3}3$
$\frac{1}{4} = 7.5$	$\frac{1}{4} = 2.5$
$\frac{1}{5} = 6$	$\frac{1}{5} = 2.$
$\frac{1}{6} = 5$	$\frac{1}{6} = 1.\dot{6}6$
Result 28.5	Result 9.50
19	19.
Error + 9.5	Error — 9.5

Pos. *Er.*

$30 \times 9.5 = 285$
$10 \times 9.5 = 95$

19) 380 (20 *Ans.*

$\frac{1}{3}$ of $20 = 6.\dot{6}$
$\frac{1}{4} = 5$
$\frac{1}{5} = 4$
$\frac{1}{6} = 3.\dot{3}$

Proof 19

Hence observe, that any question, such as this, that properly belongs to single position, may also be solved by double position.

Again, observe, that when the errors are equal and unlike, half the sum of the positions is the answer, or number sought.

5. A, B, and C, discoursing of their age, A affirmed, that he was 18 years old; B said, his age was equal to that of A, and half the age of C; and C affirmed, he was as old as both A and B: What was the age of each person? *Ans.* A 18, B 54, and C 72.

6. A father dying, left to his three sons, A, B, C, his estate in money, dividing it as follows, viz. To A he gave the half, wanting 44 l.; to B he gave a third, and 14 l. over; and to C he gave the remainder, which was 82 l. less than the share of B: What was the sum left, and what was each son's share? *Ans.* The sum left was 588 l. whereof A had 250 l. B 210 l. and C 128 l.

7. A and B had each a certain number of crowns, and said A to B, If you give me one of your crowns, I shall then have five times as many as you have behind; but said B to A, If you give me one of your crowns, then

then ſhall each of us have an equal number : How many crowns had each perſon ? *Anſ.* A had 4, and B 2.

8. A gentleman had two horſes, Cheſnut and Swift, and a ſaddle worth 50 l. which ſet on the back of Cheſnut, makes his value double that of Swift ; but the ſaddle ſet on the back of Swift, makes his value triple that of Cheſnut : What was the value of each horſe ? *Anſ.* Cheſnut 30 l. and Swift 40 l.

9. The money ſtaked by A, B, and C, playing at Hazard, was 324 crowns ; but they happening to diſagree, each ſeized as many of the crowns as he could : A got a number unknown ; B as many as A, and 15 over ; C got a fifth part of both their ſums added together : How many crowns got each ? *Anſ.* A $127\frac{1}{2}$, B $142\frac{1}{2}$, and C 54.

10. A gentleman caught a fiſh, whoſe head was 6 inches long, the tail as long as the head and half the body, the body was juſt the length of the head and tail : What was the length of the fiſh, what the length of the body, and what the length of the tail ? *Anſ.* Length of the fiſh 48 inches, length of the body 24 inches, and length of the tail 18 inches.

11. When firſt the marriage-knot was tied
 Betwixt my wife and me,
My age did hers as far exceed,
 As three times three does three ;
But after ten and half ten years,
 We man and wife had been,
Her age came up as near to mine,
 As eight is to ſixteen.
Now, Tyro, ſkill'd in numbers, ſay,
 What were our ages on the wedding-day ?

Anſwer.

Sir, Forty-five years you had been,
Your bride no more than juſt fifteen.

CHAP.

CHAP. X.

EXTRACTION of ROOTS.

IF unity be multiplied continually by any given number, the products thence arifing are called *powers of that number*; and the given number is called *the root*, or *firft power*.

Thus, if 2 be the given number, then $1 \times 2 = 2$ is the root or firft power; and $2 \times 2 = 4$ is the fqaure or fecond power; and $4 \times 2 = 8$ is the cube or third power; and $8 \times 2 = 16$ is the biquadrate or fourth power; and $16 \times 2 = 32$ is the furfolid or fifth power; and $32 \times 2 = 64$ is the fixth power, or cube fquared, &c.

The natural numbers, 1, 2, 3, &c. are fometimes placed over thefe powers, denoting the number of multiplications ufed in producing them, or fhowing what powers they are; and are called *indices* or *exponents*, as in the following fcheme.

Indices,	0,	1,	2,	3,	4,	5,	6,	7,	&c.
Powers,	1,	2,	4,	8,	16,	32,	64,	128,	&c.

The raifing any root or number given to any power required, is called *involution*; and is performed by multiplying the given root into unity continually, as taught above. But the finding the root of a given power is called *evolution*, or *extraction of roots*.

If the root of any power not exceeding the feventh power, be a fingle digit, it may be obtained by infpection, from the following table of powers.

TABLE.

T A B L E.

1st power or root.	2d power or square.	3d power or cube.	4th power or biquadrate.	5th power or surfolid.	6th power or cube squared.	7th power.
1	1	1	1	1	1	1
2	4	8	16	32	64	128
3	9	27	81	243	729	2187
4	16	64	256	1024	4096	16384
5	25	125	625	3125	15625	78125
6	36	216	1296	7776	46656	279936
7	49	343	2401	16807	117649	823543
8	64	512	4096	32768	262144	2097152
9	81	729	6561	59049	531441	4782969

I. *Extraction of the square root.*

R U L E S.

1. Divide the given number into periods of two figures, beginning at the right hand in integers, and pointing toward the left. But in decimals, begin at the place of hundreds, and point toward the right. Every period will give one figure in the root.

2. Find by the table of powers, or by trial, the nearest lesser root of the left-hand period; place the figure so found in the quot, subtract its square from the said period, and to the remainder bring down the next period for a dividual or resolvend.

3. Double the quot for the first part of the divisor; inquire how often this first part is contained in the whole resolvend, excluding the units place; and place the figure denoting the answer both in the quot and on the right of the first part; and you have the divisor complete.

4. Multiply the divisor thus completed by the figure put in the quot, subtract the product from the resolvend, and to the remainder bring down the following period for a new resolvend, and then proceed as before.

Note 1. If the first part of the divisor, with unity supposed to be annexed to it, happen to be greater than the resolvend, in this case, place 0 in the quot, and also on the right of the partial divisor; to the resolvend bring down another period, and proceed to divide as before.

Note 2. If the product of the quotient-figure into the divisor happen to be greater than the resolvend, you must go back, and give a lesser figure to the quot.

Note 3. If, after every period of the given number is brought down, there happen at last to be a remainder, you may continue the operation, by annexing periods or pairs of ciphers till there be no remainder, or till the decimal part of the quot repeat or circulate, or till you think proper to limit it.

EXAMPLE I.

Required the square root of 133225.

Square number	1̇33̇22̇5(365 root		365
	9		365
1 div. 66)	432 resolvend.		1825
	396 product.		2190
2 div. 725)	3625 resolvend.		1095
	3625 product.		133225 proof.

EXAMPLE II.

Required the square root of $54990\frac{1}{4}$.

Square

```
                 • • • •
Square number 54990.25(234.5 root.
                 4
                ---
   1 div. 43)149 resolvend.
             129 product.
             ----
  2 div. 464) 2090 resol.
              1856 prod.
              -----
 3 div. 4685) 23425 resol.
              23425 prod.
              -----
```

EXAMPLE III.

Required the square root of 72, to eight decimal places.

```
     • • • • •
   72.00000000(8.48528137 root.
   64
   ---
 164)800
     656
     -----
1688)14400
     13504
     ------
16965) 89600
       84825
       ------
169702) 477500
        339404
        ------
169704) 138096
 ....   135763
        ------
          2333
          1697
          ----
           636
           509
           ---
           127
           118
           ---
           (9)
```

After getting half of the decimal places, I work by contracted division for the other half; and obtain them with the same accuracy as if the work had been at large.

EXAMPLE IV.

Required the ſquare root of .2916

```
       .  .
     .2916(.54 root.
      25
     ----
104) 416
     416
     ---
```

EXAMPLE V.

Required the ſquare root of .001225

```
      .  .  .
     .001225(.035 root
          9
     ---------
65)···  325
        325
        ---
```

EXAMPLE VI.

Required the ſquare root of 5.875

```
        .  .
        5.875)2.423 root.
        4
        ---
44)     187
        176
        ----
482)    1150
         964
        -----
4843)   18600
        14529
        -----
        (4071)
```

The

The operation may be continued further by annexing to the remainder more periods of ciphers.

```
     2.423
     2.423
     -----
     7269
    4846
   9692
  4846
  ---------
  5.870929
      4071 rem.
  ---------
  5.875000 proof.
```

EXAMPLE VII.

Required the ſquare root of the repetend .$\dot{1}$

```
.1̇1(.3̇333 &c. root.
 9
 --
63) 211
    189
    ----
663) 2211
     1989
     -----
6663) 22211
      19989
      -----
      222211 and ſo on to infinity.
```

Here, inſtead of annexing pairs of ciphers to the remainders, I annex periods of the repetend.

EXAMPLE VIII.

Required the ſquare root of the repetend .$\dot{4}$

```
       .4̇4(.6̇66 &c. root.
        36
 126)  844
        756
1326)  8844
        7956
        88844
```

Note. No repeating digit, except .1̇ and 4̇ have for their roots a pure single repetend.

EXAMPLE IX.

Required the square root of 1320.1̇

```
        1320.1̇(36.3̇3 &c. root.
         9
    66)420
        396
   723) 2411
        2169
  7263) 24211
        21789
        242211
```

EXAMPLE X.

Required the square root of the circulate 138.518; or, which is the same thing, of 13,8.51,

13,8.51,

```
          ·  ·  ·
         13,8.51,(11.769389 root.
         1
        ——
   21)  38
        21
       ————
  227)1751
       1589
       —————
 2346) 16285
        14076
       ——————
23529) 220918
        211761
        ——————
 23538) 9157
   ···  7061
        ————
        2096
        1883
        ————
         213
         211
        ————
         (2)
```

Here I annex to the remainders periods of the circulating figures.

I continue the work till I have three decimal places in the root; and then, by contracted divifion, I obtain three more.

If the fquare root of a vulgar fraction be required, find the root of the given numerator for a new numerator, and find the root of the given denominator for a new denominator. Thus, the fquare-root of $\frac{4}{9}$ is $\frac{2}{3}$, and the root of $\frac{16}{25}$ is $\frac{4}{5}$; and thus the root of $\frac{25}{4}$ ($= 6\frac{1}{4}$) is $\frac{5}{2} = 2\frac{1}{2}$.

But if the root of either the numerator or denominator cannot be extracted without a remainder, reduce the vulgar fraction to a decimal, and then extract the root, as in Example 4. and 5.

The extraction of roots may be conducted in the way of vulgar fractions; but then the operation becomes involved and troublefome. The following example done both ways will fhew the fuperior excellence of decimals.

Required the fquare root of $20\frac{1}{4}$

By

By vulgar fractions.	Decimally.
$20\frac{1}{4}(4\frac{1}{2}$ root.	$20.25(4.5$
16	16
$8\frac{1}{2})\ 4\frac{1}{4}$	$85)425$
$4\frac{1}{4}$	425
(0)	(0)

The method of extracting the ſquare root is founded on Euclid, book 2. prop. 4. where it is demonſtrated, That if any line, and conſequently any number, be divided into two parts, the ſquare of the whole line or number will be equal to the ſquares of the two parts, together with twice a rectangle or product under the parts. Thus, if the number 10 be divided into two parts, ſuppoſe 6 and 4, the ſquare of 10 will be equal to $6 \times 6 = 36 + 4 \times 4 = 16$, $+$ twice $6 \times 4 = 48$; for $36 + 16 + 48 = 100 = 10 \times 10$.

In order to ſhew the reaſon of the rules, we muſt have recourſe to algebra, in which a letter, as a, is put to repreſent any number at pleaſure; and aa denotes the ſquare of a, and aaa its cube; ay denotes a product, and $2ay$ denotes a double product; and the factor into which any letter is multiplied, is called its *coefficient*.

Now, let the firſt two periods of the given ſquare number be conſidered as a ſquare number by themſelves; and as every period gives a figure to the root, their root will conſiſt of two figures, or two parts; call the firſt a, and the other y; then $a + y$ is their root complete; and by Euclid ii. 4. or by algebraic multiplication, $aa + 2ay + yy$ will be equal to the firſt two periods of the ſquare number given; and when aa, the ſquare of the firſt part of the root, is ſubtracted from the firſt period, there will remain $2ay + yy$; and to find y, the other part of the root, I divide by its factor or coefficient, viz. by $2a + y$; that is, I find a diviſor by doubling the quot, and annexing to it the next quotient-figure.

N. B. The figure of the quot denoted by a, in regard another figure is to follow, is conſidered as in the place of

of tens, having a cipher on the right, and consequently so has the first part of the divisor; and therefore the annexing the quotient-figure to the said partial divisor is the same thing in this case as adding it.

If there be more periods of the given square number, the two figures now in the quot are to be esteemed the first part of the root, and called *a*; and by repeating the operation, as above directed, the other part *y* is found. And if there be still more periods, call the three figures now in the quot *a*, then repeat the operation, and find *y* as before. And thus proceed till every period is brought down.

MORE EXAMPLES.

1. There are two numbers, whereof the lesser is 3456, their difference is 293392: What is the greater, and what the square root of their sum? *Ans.* The greater is 296848, and the square root of their sum is 548.

2. There is a round floor, whose content is, 45.938271604 square feet, or 45 feet 11 inches 3 lines and 4 fourths: Required the side of a square floor equal to it in area? *Ans.* 6.7 feet, or 6 feet 9 inches 4 lines. See duodecimals in decimal practice, Ex. 3.

3. There are three fields, whereof the first contains 100 acres 3 roods and 36 poles; the second contains 118 acres 2 roods and 24 poles; and the third contains 122 acres 2 roods and 20 poles: Required the side of a square field that shall be equal to all the three in area? *Ans.* The side of the square field is 234 poles nearly, or 58 chains and 2 poles.

4. A round malt-kiln, whose diameter is 12 feet, is to be enlarged, so as to hold three times as much malt as formerly: Required the new diameter? *Ans.* 20.78 feet. The areas of circles are to one another as the squares of their diameters, by Euclid xii. 2.; multiply therefore the square of the given diameter by 3, and extract the square root of the product.

5. A tower is surrounded by a ditch 40 feet wide, and a scaling ladder 50 feet long will reach from the

outſide of the ditch to the top of the tower: Required the height of the tower? *Anſ.* 30 feet. The ſquare of the hypotenuſe, or longeſt ſide of a right-angled triangle, is equal to the ſum of the ſquares of the other two ſides, by Euclid i. 47.; and therefore, from the ſquare of 50 ſubtract the ſquare of 40, and the ſquare root of the remainder is the height of the tower.

6. Three towns, A, B, C, are ſo ſituate, that B lies 80 miles ſouth of A, and C 60 miles weſt of A: What is the diſtance between B and C? *Anſ.* 100 miles.

II. *Extraction of the cube root.*

R U L E S.

1. Divide the given number into periods of three figures, beginning at the right hand in integers, and pointing toward the left. But in decimals, begin at the place of thouſands, and point toward the right. The number of periods ſhews the number of figures in the root.

2. Find by the table of powers, or by trial, the neareſt leſſer root of the left-hand period; place the figure ſo found in the quot; ſubtract its cube from the ſaid period; and to the remainder bring down the next period for a dividual or reſolvend.

The diviſor conſiſts of three parts, which may be found, as follows.

3. The firſt part of the diviſor is found thus: Multiply the ſquare of the quot by 3, and to the product annex two ciphers; then inquire how often this firſt part of the diviſor is contained in the reſolvend, and place the figure denoting the anſwer in the quot.

4. Multiply the former quot by 3, and the product by the figure now put in the quot; to this laſt product annex a cipher; and you have the ſecond part of the diviſor. Again, ſquare the figure now put in the quot for the third part of the diviſor; place theſe three parts under one another, as in addition; and their ſum will be the diviſor complete.

5. Multiply

5. Multiply the divifor, thus completed, by the figure laft put in the quot, fubtract the product from the refolvend, and to the remainder bring down the following period for a new refolvend, and then proceed as before.

Note 1. If the firft part of the divifor happen to be equal to or greater than the refolvend, in this cafe, place o in the quot, annex two ciphers to the faid firft part of the divifor, to the refolvend bring down another period, and proceed to divide as before.

Note 2. If the product of the quotient-figure into the divifor happen to be greater than the refolvend, you muft go back, and give a leffer figure to the quot.

Note 3. If after every period of the given number is brought down, there happen at laft to be a remainder, you may continue the operation by annexing periods of three ciphers till there be no remainder, or till you have as many decimal places in the root as you judge neceffary.

E X A M P L E I.

Required the cube root of 12812904.

```
                       .  .  .
      Cube number  12812904(234 root.
                    8
                   ---
 1ft part 1200 ⎤      )4812 refolvend.
 2d part   180 ⎬
 3d part     9 ⎦
          ----
1 divifor 1389 × 3 = 4167 product.
---------------------------------------------
 1ft part 158700 ⎤     )645904 refolvend.
 2d part    2760 ⎬
 3d part      16 ⎦
          ------
2 divifor 161476 × 4 = 645904 product.
                       ------
```

P R O O F.

	234	Square	54756
	234		234
	936		219024
	702		164268
	468		109512
Square	54756	Cube	12812904

The reaſon of the rules will appear, if we take the firſt two periods of the given cube number, and conſider them as a cube number by themſelves, whoſe root will conſiſt of two figures or parts, which call a and y; and then, by algebraic multiplication, the firſt two periods of the cube number will be equal to $aaa + 3aay + 3ayy + yyy$. Now, when aaa, the cube of the firſt part of the root, is ſubtracted from the left-hand period, there remains $3aay + 3ayy + yyy$; and to find y, the other part of the root I divide by its coefficient, viz. by $3aa + 3ay + yy$; and as the diviſor conſiſts of three parts, I begin with the firſt part, viz. $3aa$, and try how often it is contained in the reſolvend; and then, by the help of the figure that goes to the quot, I make up the other two parts, as follows.

1ſt part	1200	$= 3aa$	$= 3 \times 2 \times 2$
2d part	180	$= 3ay$	$= 3 \times 2 \times 3$
3d part	9	$= yy$	$= 3 \times 3$
1 diviſor	1389		

The reaſon of annexing one cipher for every a, viz. two ciphers in the firſt part of the diviſor, and one in the ſecond, is becauſe the firſt figure of the root 2, another figure being to follow on the right, is really and truly 20.

If, as in the above example, there be more periods of the given cube number, the two figures now in the quot are to be eſteemed the firſt part of the root, and called a;

a; and by repeating the operation for a new divisor, the other part y, may be found. Thus,

$$\begin{array}{lrcl} \text{1st part} & 158700 & = 3aa = & 3 \times 23 \times 23 \\ \text{2d part} & 2760 & = 3ay = & 3 \times 23 \times 4 \\ \text{3d part} & 16 & = yy = & 4 \times 4 \\ \hline \text{2 divisor} & 161476 & & \end{array}$$

In the first and second part of the divisor, the ciphers are annexed for the reason assigned above.

If there be still more periods in the cube number, call the three figures now in the quot a, repeat the operation, and find y as before; and thus proceed till every period is brought down.

The reason of dividing the given number into periods of two figures in the square, of three in the cube, of five in the sursolid, &c. and of there being as many figures in the root as there are periods, arises from the nature of involution; and will appear by resolving any number into its constituent parts, and raising them separately to the second, third, fourth, fifth power, &c. Let the number be $7842 = 7000 + 800 + 40 + 2$. Now, 7 has three places after it; but in the square it will have six, and in the cube nine. Again, 8 has two places after it; but in the square it will have four, and in the cube six, &c.

$$\begin{array}{lr} \text{Thus} & 7\,000 \\ & 7\,000 \\ \hline \text{Square} & 49{,}00{,}00{,}00 \\ & 7\,000 \\ \hline \text{Cube} & 343{,}000{,}000{,}000 \end{array} \qquad \begin{array}{lr} \text{And thus} & 8\,00 \\ & 8\,00 \\ \hline \text{Square} & 64{,}00{,}00 \\ & 8\,00 \\ \hline \text{Cube} & 512{,}000{,}000 \end{array}$$

Hence it is plain, that the square of 7842 will contain four periods of two figures, and its cube will contain four periods of three figures; and so the number of periods will be equal to the number of figures in the root. And the square and cube of 842 will in like manner contain three periods each.

EX-

EXAMPLE II.

Required the cube root of $28\frac{3}{4}$.

```
                              .   .   .
                            28.750000(3.06 root.
                            27
 270000┐                    ) 1750000 resolv.
   5400 >
     36┘
Div. 275436 × 6 = 1652616 prod.
                  97384 rem.
```

PROOF.

```
      3.06              Sq. 9.3636
      3.06                    3.06
      1836                  561816
      918                 280908
Sq.  9.3636               28.652616
                              97384 rem.
                          28.750000 cube.
```

EXAMPLE III.

Required the cube root of .000485613.

```
                             .   .   .
                           .000485613(.078 root.
                                 343
 14700┐                    )··· 142613 resolv.
  1680 >
    64┘
Div. 16444 × 8 = 131552 prod.
                 11061 rem.
```

EX-

EXAMPLE IV.

Required the cube root of 7584.

7584(19.64 root.
1

$$\left.\begin{array}{r}300\\270\\81\end{array}\right\}\qquad)6584 \text{ resolv.}$$

1 div. 651 × 9 = 5859 prod.

$$\left.\begin{array}{r}108300\\3420\\36\end{array}\right\}\qquad)725000 \text{ resolv.}$$

2 div. 111756 × 6 = 670536 prod.

$$\left.\begin{array}{r}11524800\\23520\\16\end{array}\right\}\qquad)54464000 \text{ resolv.}$$

3 div. 11548336 × 4 = 46193344 prod.

Rem. 8270656

If the cube root of a vulgar fraction be required, find the cube root of the given numerator for a new numerator, and the cube root of the given denominator for a new denominator. Thus, the cube root of $\frac{8}{27}$ is $\frac{2}{3}$, and the cube root of $\frac{27}{64}$ is $\frac{3}{4}$; and thus the cube root of $\frac{125}{8}$ ($= 15\frac{5}{8}$) is $\frac{5}{2} = 2\frac{1}{2}$.

But if the root of either the numerator or denominator cannot be extracted without a remainder, reduce the vulgar fraction to a decimal, and then extract the root, as in Ex. 3.

MORE EXAMPLES.

1. There are two numbers, whereof the greater is 579890752, their difference is 1152: What is the lesser, and what the cube root of their sum? *Ans.* The lesser

leſſer is 2579889600, and the cube root of their ſum is 1728.

2. The content of an oblong cellar is 1953.125 cubic feet : Required the ſide of a cubical cellar that ſhall contain juſt as much. *Anſ.* 12.5 feet.

3. There are three boxes, the content of one is 10000 ſolid inches, of another 16656, and of the third 20000 : Required the ſide of a cubical box that ſhall contain as much as all the three. *Anſ.* 36 inches, or 3 feet.

4. Required the ſide of a cubical veſſel that will contain 80 wine gallons. *Anſ.* 26.43 inches.

231 cubic inches in a wine gallon.
80
———
18480 whoſe cube root is 26.43 inches.

5. A ſhip's length is 50 feet, breadth 20, and depth of the hold 10 : Required the dimenſions of a like veſſel triple her burden.

L. 50, its cube 125000 × 3 = 375000, its cube root 71.54
B. 20, its cube 8000 × 3 = 24000, its cube root 29.24
D. 10, its cube 1000 × 3 = 3000, its cube root 14.44

6. A braſs bullet of 5 inches diameter weighs 20 lb. : What is the diameter of a like bullet that weighs 160 lb.? *Anſ.* 10 inches.

	lb.			*lb.*
As	20 :	5 × 5 × 5	: :	160
	4 :	5 × 5	: :	160
	1 :	25	: :	40 : 1000, whoſe cube root is 10 inches.

III. *Extraction of the biquadrate root.*

R U L E.

Extract the ſquare root of the given number; and again

gain extract the square root of the root so found, and the last of these roots is the root sought.

EXAMPLE.

Required the biquadrate root of 5308416

```
          . . . .   . .
          5308416  (2304(48 root.
          4         16
         ____      _____
      43)130    88)704
          129       704
         _______   _____
  4604)    18416
           18416
          _______
```

If, in the first extraction, there happen to be a remainder, continue the operation, by annexing pairs of ciphers, till you have twice as many decimal places in the square or first root, as you propose to have in the last root.

IV. *Extraction of the root of the fifth power, or sursolid.*

RULES.

1. Divide the given number into periods of five figures, find the nearest lesser root of the left-hand period, put the figure so found in the quot, subtract its fifth power, and to the remainder bring down the next period for a resolvend.

2. Put $a + y$ for the root, and then the sursolid or fifth power will be $aaaaa + 5aaaay + 10aaayy + 10aayyy + 5ayyyy + yyyyy$. Now, $aaaaa$ being already subtracted, there remains the other five parts; and to find y, I divide by its coefficient, viz. by $5aaaa + 10aaay + 10aayy + 5ayyy + yyyy$; that is, I try how often $5aaaa$ is contained in the resolvend; and, by the help of the

 quotient-

quotient-figure, I make up the other four parts of the divisor.

EXAMPLE.

Required the sursolid root of 33554432

$$
\begin{array}{rcl}
 & & \dot{3}35\dot{5}4432(32 \text{ root.} \\
 & & 243 \\
\hline
 & &)\,9254432 \text{ resolv.} \\
4050000 & = & 5aaaa \\
540000 & = & 10aaay \\
36000 & = & 10aayy \\
1200 & = & 5ayyy \\
16 & = & yyyy \\
\hline
\text{Divisor } 4627216 \times 2 & = & 9254432 \text{ prod.} \\
\hline
 & & (0)
\end{array}
$$

V. *Extraction of the root of the sixth power, or cube squared.*

RULE.

Extract the square root of the given number, and then extract the cube root of that root, the last is the root sought. Or, first extract the cube root, and then extract the square root of that root.

EXAMPLE.

Required the root of 191102976, being the sixth power.

$$\dot{1}9\dot{1}1\dot{0}2\dot{9}7\dot{6}$$

191102976

```
 ·  ·  ·  ·  ·     ·  ·
191102976  (13824(24 root.
1             8
------      ------
23)·91      1200)5824 resolv.
   69        240
   ----       16
268)2210   ------
    2144   1456 × 4 = 5824 prod.
    -----            ------
2762)  6629           (0)
       5524
      ------
27644)110576
      110576
     --------
```

VI. *Extraction of the root of the seventh power.*

R U L E.

Put $a+y$ for the root, and the seventh power will be $aaaaaaa + 7aaaaaay + 21aaaaayy + 35aaaayyy + 35aaayyyy + 21aayyyyy + 7ayyyyyy + yyyyyyy$, by the aid of which proceed as in extracting the root of the fifth power.

E X A M P L E.

Required the root of 3404825447, being the seventh power.

$$
\begin{array}{rcl}
 & & \dot{3}404\dot{8}25447(23 \text{ root.} \\
 & & \underline{128} \\
 & &)\underline{2124825447} \text{ resolv.} \\
448000000 & = & 7aaaaaa \\
201600000 & = & 21aaaaay \\
50400000 & = & 35aaaayy \\
7500000 & = & 35aaayyy \\
680400 & = & 21aayyyy \\
34020 & = & 7ayyyyy \\
\underline{729} & = & yyyyyy \\
\text{Divisor } 708275149 \times 3 & = & \underline{2124825447} \text{ prod.} \\
 & & (0)
\end{array}
$$

VII. *Extraction of the root of the eighth power.*

R U L E.

Extract the square root of the given number continually till you have three roots; the last of these is the root sought.

Thus, let 178579390489 6 be the eighth power; by extracting the square root you get the biquadrate or fourth power, viz. 1336336; and by extracting the square root of the biquadrate, you get the square or second power, viz. 1156, whose square root is 34, the root sought.

VIII. *Extraction of the root of the ninth power.*

R U L E.

Extract the cube root of the given number, and you have the cube or third power, whose cube root is the root sought.

Thus, let 5159780352 be the ninth power; by extracting the cube root you get the cube or third power, viz. 1728, whose cube root 12 is the root sought.

Universally, whatever the given power be, put $a+y$ for

for the root, and by involution raiſe $a + y$ to the power of the given number; then, with this as your guide or canon, extract the root in the manner preſcribed and exemplified in the extraction of the root of the fifth and ſeventh powers.

But if the index of the given power be a multiple of 2, the work may be rendered eaſier: for by extracting the ſquare root of the given number, you obtain a power whoſe index is one half of the index of the given power. Thus, by extracting the ſquare root of the tenth power, you have the fifth power; and the ſquare root of the twelfth power is the ſixth power, &c.

Again, if the index of the given power be a multiple of 3, by extracting the cube root you obtain a power whoſe index is one third of the index of the power given. Thus the cube root of the ninth power is the cube or third power; and the cube root of the twelfth power is the biquadrate or fourth power, &c.

Involution is directly contrary to extraction or evolution; and therefore, if a ſquare number be ſquared, it will give the biquadrate or fourth power; and if a biquadrate be ſquared, it will give the eighth power. Again, if a cube number be cubed, it will give the ninth power; and if the biquadrate be cubed, it will give the twelfth power.

CHAP. XI.

PROGRESSION.

THE comparing of numbers with one another is called *Comparative Arithmetic*; and in comparing any two numbers together, the firſt number, or the number compared, is called the *antecedent*, and the other number with which it is compared, is called the *conſequent*.

In comparing two numbers, we may, by ſubtracting the leſſer from the greater, find their difference; and the difference thus found is called the *arithmetical ratio*

tio of the two numbers; and if two or more pairs of numbers have all the ſame ratio or difference, ſuch numbers are ſaid to be in *arithmetical proportion*; and the ratio in reference to the ſeveral proportional numbers, is uſually called the *common difference*; the firſt and laſt term are called *extremes*, and the intermediate ones are denominated *means*.

If a rank of numbers, without interruption, have all the ſame difference, the proportion is ſaid to be *continued* or *continual*; and ſuch a rank is called an *arithmetical progreſſion*, or an *arithmetical ſeries*; ſuch as, 2, 4, 6, 8, 10, 12.

But if, in the rank of proportional numbers, there be a breach or chaſm, the proportion is ſaid to be *diſcontinued, disjunct*, or *interrupted*; ſuch as, 3, 5, 7, —— 19, 21, 23.

Again, in comparing two numbers, we may find how often the one contains the other, by dividing the antecedent by the conſequent; and the quot thence ariſing is called the *geometrical ratio* of the two numbers; and if two or more pairs of numbers have all the ſame ratio, ſuch numbers are ſaid to be in *geometrical proportion*. The firſt and laſt of the proportional numbers are called *extremes*, and the other terms are denominated *means*.

If a rank of numbers, without breach or chaſm, have all the ſame ratio, the proportion is ſaid to be *continued* or *continual*; and ſuch a rank is called a *geometrical progreſſion*, or a *geometrical ſeries*; ſuch as, 2 : 4 : 8 : 16 : 32.

But if, in the rank of proportional numbers, there be a chaſm, the proportion is ſaid to be *diſcontinued, diſjunct*, or *interrupted*; ſuch as, 12 : 6 :: 30 : 15.

I. *Arithmetical Progreſſion*.

An arithmetical progreſſion, or ſeries, is formed either from 0, or from ſome number aſſumed as the firſt term, by continual addition or ſubtraction, and is either increaſing or decreaſing.

Thus, 0, 3, 6, 9, 12, 15, 18, is an arithmetical progreſſion

greſſion or ſeries, formed from o, aſſumed as the firſt term, and increaſing by the continual addition of 3; and ſuch a ſeries may be continued upward to infinity.

And 14, 12, 10, 8, 6, 4, 2, is an arithmetical progreſſion, or ſeries, formed from 14, aſſumed as the firſt term, and decreaſing by the continual ſubtraction of 2; but ſuch a ſeries can be continued downward only till the laſt term becomes equal to or leſs than the common difference.

In an arithmetical progreſſion, or ſeries, five things occur to be conſidered.

I. The leaſt term. } Extremes.
II. The greateſt term. }
III. The number of terms.
IV. The common difference.
V. The ſum of all the terms.

The properties of numbers in arithmetical progreſſion are various and numerous; but the chief, or moſt uſeful, which alone we propoſe to take notice of in this place, are explained in the following theorems.

THEOREM I.

Any term of an arithmetical ſeries is equal to the ſum of the leaſt term, and the product of the common difference into the number of terms before or after it.

Thus, in the increaſing ſeries, 2, 5, 8, 11, 14, 17, the term $17 = 2 + 3 \times 5 = 2 + 15$.

Again, in the decreaſing ſeries, 18, 16, 14, 12, 10, 8, 6, 4, 2, the term $18 = 2 + 8 \times 2 = 2 + 16$.

The reaſon is plain; becauſe every term ſubſequent or prior to the leaſt is made up by the continual addition or ſubtraction of the common difference.

Hence the product of the common difference into the number of terms minus unity, added to the leaſt term gives the greateſt, or ſubtracted from the greateſt term leaves the leaſt.

THEO-

THEOREM II.

In an arithmetical ſeries, conſiſting of three, five, or any odd number of terms, the double of the middle term is equal to the ſum of the two extremes, or to the ſum of any two means equally diſtant from the ſaid middle term.

Thus, in the ſeries, 1, 2, 3, 4, (5), 6, 7, 8, 9, the middle term 5 doubled is $10 = 1 + 9$, the two extremes, or $= 2 + 8$, or $3 + 7$, or $4 + 6$.

The reaſon is obvious: for of two terms equally diſtant from the middle term, the one exceeds the middle term juſt as much as the other is below it.

COROLLARIES.

1. Hence an arithmetical mean betwixt two given extremes is found by taking half the ſum of the extremes. Thus, the mean betwixt 6 and 10 is 8; for $6 + 10 = 16$, and 2)16(8. So 6, 8, 10, are in arithmetical proportion.

2. Hence alſo, to an extreme and mean given, a third proportional, or the other extreme, is found by ſubtracting the given extreme from the mean doubled. Thus, to 3, 6, given, the third proportional is 9; for $2 \times 6 = 12$, and $12 - 3 = 9$: ſo 3, 6, 9, are in arithmetical proportion. In like manner, to 10, 7, given, the third proportional is 4; for $2 \times 7 = 14$, and $14 - 10 = 4$: ſo 10, 7, 4, are arithmetical proportionals.

THEOREM III.

In an arithmetical ſeries, conſiſting of four, ſix, or any even number of terms, the ſum of the extremes is equal to the ſum of the two middle terms, or to the ſum of any two means equally diſtant from the extremes.

Thus, in the ſeries, 3, 5, 7, 9, 11, 13, the ſum of $3 + 13 = 7 + 9$, or $= 5 + 11$.

The reaſon is plain: for if betwixt the two middle terms in the ſeries, a middle term be inſerted, the ſum of the extremes, as alſo the ſum of every pair of equidiſtant means,

means, will, by Theorem II. be equal to the double of the inserted middle term; and things equal to one and the same thing are equal to one another.

COROLLARIES.

1. Hence, to an extreme and two means given, the other extreme, or fourth proportional, is found, by subtracting the given extreme from the sum of the two given means. Thus, to 4, 6, 8, the fourth proportional is 10; for $6+8=14$, and $14-4=10$; so 4, 6, 8, 10, are in arithmetical proportion. And this takes place though the proportion be disjunct. Thus, to 18, 14,—— 6, the fourth proportional is 2; for $14+6=20$, and $20-18=2$; so 18, 14,—— 6, 2, are in arithmetical proportion disjunct.

2. Hence likewise, when two extremes and a mean are given, the other mean is found by subtracting the given mean from the sum of the two extremes. Thus, to 13, 9, 7, the other mean is 11; for $13+7=20$, and $20-9=11$; so 13, 11, 9, 7, are arithmetical proportionals. This also takes place, though the proportion be disjunct. Thus, to 9, 15,——36, the other mean is 30; for $9+36=45$, and $45-15=30$; so 9, 15,—— 30, 36, are in arithmetical proportion disjunct.

THEOREM IV.

In an arithmetical series, the difference of the extremes, divided by the common difference, gives a quot, which, increased by unity, is the number of terms.

Thus, in the series, 2, 4, 6, 8, 10, 12, 14, the difference of the extremes 2 and 14 is 12, and 12, divided by the common difference 2, quotes 6, and $6+1=7$, the number of terms.

The reason is obvious: for the difference of the extremes is made up by a repetition of the common difference as often as there are terms save one; that is, the difference of the extremes is a product of the common difference into the number of terms minus unity.

THEOREM V.

In an arithmetical ſeries, the difference of the extremes, divided by the number of terms minus unity, quotes the common difference.

Thus, in the ſeries, 3, 5, 7, 9, 11, 13, the difference of the extremes 3 and 13 is 10, and 10 divided by the number of terms minus unity, viz. 5, quotes 2, the common difference.

The reaſon is the ſame here as in the former theorem.

THEOREM VI.

In an arithmetical ſeries, the ſum of all the terms may be made out ſeveral ways, as follows.

1. The ſum of the extremes, multiplied by the number of terms, gives a product, which divided by 2, quotes the ſum of all the terms.

Thus, in the ſeries, 3, 6, 9, 12, 15, 18, 21, the ſum of 3 and 21 is 24; and $24 \times 7 = 168$; and 2)168(84, the ſum of all the terms.

The reaſon will appear by inverting the ſeries, and adding each equidiſtant pair of means, as follows.

$$\begin{array}{ccccccc} 3, & 6, & 9, & 12, & 15, & 18, & 21 \\ 21, & 18, & 15, & 12, & 9, & 6, & 3 \\ \hline 24, & 24, & 24, & 24, & 24, & 24, & 24 \end{array}$$

Now, as the ſum of every pair is 24, and the number of terms 7, it is plain, the product of 7 into 24 will be double the ſum of the ſeries.

2. The ſum of the extremes multiplied by half the number of terms, gives a product equal to the ſum of all the terms.

Thus, in the ſeries, 12, 10, 8, 6, 4, 2, the ſum of 12 and 2 is 14, and $14 \times 3 = 42$, the ſum of all the terms.

The reaſon is the ſame as before.

3. The half ſum of the extremes multiplied by the number of terms, gives a product equal to the ſum of all the terms.

Thus,

Thus, in the ſeries, 1, 4, 7, 10, 13, 16, 19, the half ſum of 1 and 19 is 10, and $10 \times 7 = 70$, the ſum of all the terms.

The reaſon is the ſame as above.

4. When a ſeries conſiſts of an odd number of terms, the middle term multiplied by the number of terms, gives a product equal to the ſum of all the terms.

Thus, in the ſeries, 3, 6, (9), 12, 15, the middle term $9 \times 5 = 45$, the ſum of all the terms.

The reaſon is plain; becauſe the middle term, by Theorem II. is equal to half the ſum of the extremes.

5. In a ſeries of the natural numbers, 1, 2, 3, 4, 5, &c. the greateſt given term multiplied into the next greater, gives a product, which divided by 2, quotes the ſum of the whole ſeries.

Thus, in the ſeries, 1, 2, 3, 4, 5, 6, 7, 8, 9, the greateſt given term, 9, multiplied into the next greater, 10, gives 90; and 2)90(45, the ſum of the whole ſeries.

The reaſon is, becauſe the greateſt term is, in this caſe, the number of terms, and the next greater term is the ſum of the extremes; therefore, &c.

6. In a ſeries of the natural odd numbers, 1, 3, 5, 7, &c. the ſquare of the number of terms is equal to the ſum of the whole ſeries.

Thus, in the ſeries, 1, 3, 5, 7, 9, 11, 13, the number of terms is 7, and $7 \times 7 = 49$, the ſum of the whole ſeries.

The reaſon is, becauſe, in this caſe, the half ſum of the extremes is always equal to the number of terms.

7. In a ſeries of the natural even numbers, 2, 4, 6, 8, &c. the number of terms multiplied into the number of terms plus unity, gives a product equal to the ſum of the whole ſeries.

Thus, in the ſeries, 2, 4, 6, 8, 10, 12, the number of terms is 6, and $6 \times 7 = 42$, the ſum of the whole ſeries.

The reaſon is, becauſe, in this caſe, the half ſum of the extremes always exceeds the number of terms by unity.

THEOREM VII.

In an arithmetical ſeries, the ſum of all the terms divided by the number of terms, quotes half the ſum of the extremes; and half the product of the common difference multiplied into the number of terms minus unity, is equal to half the difference of the extremes.

The reaſon of the firſt part of the propoſition is evident; for, by Theorem VI. 2. the ſum of the extremes multiplied into half the number of terms, gives a product equal to the ſum of all the terms; and conſequently the ſum of all the terms divided by half the number of terms, quotes the ſum of the extremes; and doubling the diviſor, that is, dividing by the number of terms, the quot will be half the ſum of the extremes.

The ſecond part of the propoſition is likewiſe plain: for, by Theorem V. the difference of the extremes, divided by the number of terms minus unity, quotes the common difference; but the quote multiplied into the diviſor, produces the dividend; which, in this caſe, divided by 2, quotes half the difference of the extremes.

Hence the ſum of a ſeries, the number of terms, and the common difference being given, the extremes may be found: for the theorem gives their half ſum, and half difference; and the half difference of two quantities added to half their ſum, gives the greater quantity; and the half difference ſubtracted from half their ſum, leaves the leſſer quantity.

Of the five things that occur to be conſidered in an arithmetical ſeries, if any three be given, the other two may be found; the exemplification whereof at large would open a wide field; but we propoſe only to give a brief ſpecimen, in a few problems, whoſe ſolutions are founded upon, and flow directly from the above theorems.

PROB. I.

Given the greateſt term, the number of terms, and common difference, to find the leaſt term; that is, Given II. III. IV. to find I.

RULE.

RULE.

Multiply the common difference into the number of terms minus unity, ſubtract the product from the greateſt term, and the remainder is the leaſt term, by Theorem I.

EXAMPLE I.

A man travels 6 days, and increaſes each day's journey by 4 miles; his laſt day's journey was 40 miles: What was the firſt? *Anſ.* 20 miles.

$4 \times 5 = 20$, and $40 - 20 = 20$ miles. *Anſ.*
Series 20, 24, 28, 32, 36, 40, complete.

EXAMPLE II.

A man takes out of his pocket, at eight ſeveral times, ſo many ſeveral numbers of ſhillings, every one exceeding the former by 6; the laſt was 54: What was the firſt? *Anſ.* 12 ſhillings.

$6 \times 7 = 42$, and $54 - 42 = 12$ s. *Anſ.*

PROB. II.

Given the leaſt term, the number of terms, and common difference, to find the greateſt term; that is, Given I. III. IV. to find II.

RULE.

Multiply the common difference into the number of terms minus unity, add the product to the leaſt term, and the ſum is the greateſt term, by Theorem I.

EXAMPLE I.

There is an arithmetical ſeries conſiſting of 18 terms or places; the firſt term is 4, the common difference 12: What is the greateſt term? *Anſ.* 208.

$12 \times 17 = 204$, and $204 + 4 = 208$. *Anſ.*

EX-

EXAMPLE II.

A man had 12 children; the youngeſt was three years old, and the common difference of their ages was 4 years: What was the age of the eldeſt? *Anſ.* 47 years.

$4 \times 11 = 44$, and $44 + 3 = 47$ years. *Anſ.*

PROB. III.

Given the extremes, and common difference, to find the number of terms; that is, given I. II. IV. to find III.

RULE.

Divide the difference of the extremes by the common difference, and the quot plus unity is the number of terms, by Theorem IV.

EXAMPLE I.

A ſetting out on a journey, travels 12 miles the firſt day, and increaſes every day's journey by four miles, till at laſt he travelled 64 miles in one day: How many days did he travel? *Anſ.* 14 days.

$64 - 12 = 52$, and 4)52(13, and $13 + 1 = 14$ days. *Anſ.*

EXAMPLE II.

The youngeſt of a large family was four years old, the eldeſt 40, and the common difference of the childrens ages was 3 years: How many children were there? *Anſ.* 13.

$40 - 4 = 36$, and 3)36(12, and $12 + 1 = 13$ *Anſ.*

PROB. IV.

Given the extremes, and number of terms, to find the common difference; that is, given I. II. III. to find IV.

RULE.

RULE.

Divide the difference of the extremes by the number of terms minus unity, and the quot is the common difference, by Theorem V.

EXAMPLE I.

A man had 12 ſons, whoſe ages were in arithmetical progreſſion; the youngeſt was 2 years old, and the eldeſt 35: What was the common difference of their ages? *Anſ.* 3 years.

$35 - 2 = 33$, and 11)33(3 years. *Anſ.*

EXAMPLE II.

A gentleman diſtributes a ſum of money in arithmetical progreſſion among 20 beggars; to the firſt he gave 3 ſhillings, and to the laſt 41 ſhillings: What was the common difference? *Anſ.* 2 ſhillings.

$41 - 3 = 38$, and 19)38(2 s. *Anſ.*

PROB. V.

Given the extremes, and number of terms, to find the ſum of the ſeries; that is, given I. II. III. to find V.

The ſeveral ſolutions of this problem aſſigned in Theorem VI. ſupply the place of rules; and to theſe we ſhall conſtantly refer.

EXAMPLE I.

13 perſons give their charity to a poor man in arithmetical progreſſion; the firſt gave 2 d. the laſt 26 d.: How much did the poor man get? *Anſ.* 182 d. or 15 s. 2 d. Theorem VI. 1.

$2 + 26 = 28$, and $28 \times 13 = 364$, and 2)364(182 d. *Anſ.*

EXAMPLE II.

Placing 100 eggs in a ſtraight line, at a yard's diſtance

ſtance one from another, and the firſt a yard from a baſket: How far muſt one travel to bring the eggs, one by one, into the baſket? *Anſ.* 10100 yards, or 5 English miles and 1300 yards. Theorem VI. 2.

$2 + 200 = 202$, and $202 \times 50 = 10100$ yards. *Anſ.*

EXAMPLE III.

A man bought 12 growing trees, the firſt for 2 s. the laſt for 40 s.; and their prices being in arithmetical progreſſion, what paid he for the whole? *Anſ.* 252 s. or 12 l. 12 s. Theorem VI. 3.

$2 + 40 = 42$, and 2)42(21, and $21 \times 12 = 252$ s. *Anſ.*

EXAMPLE IV.

A perſon completes a journey in 13 days, the number of miles travelled each day being in arithmetical progreſſion, the ſeventh day he travelled 22 miles: What was the length of his journey? *Anſ.* 286 miles. Theorem VI. 4.

$22 \times 13 = 286$ miles. *Anſ.*

EXAMPLE V.

How many ſtrokes does a clock ſtrike in one revolution of the index, viz. in 12 hours? *Anſ.* 78 ſtrokes. Theorem VI. 5.

$12 \times 13 = 156$, and 2)156(78 ſtrokes. *Anſ.*

EXAMPLE VI.

A butcher bought a dozen of fat lambs, and was to pay 1 ſhilling for the firſt, 3 for the ſecond, and 5 for the third, &c. ſtill advancing 2 ſhillings more for every following one: What did the 12 lambs coſt him? *Anſ.* 144 ſhillings, or 7 l. 4 s. Theorem VI. 6.

$12 \times 12 = 144$ ſhillings. *Anſ.*

EXAMPLE VII.

A man buys 10 horfes, and is to pay 2 l. for the firft, 4 for the fecond, 6 for the third, &c. ftill paying 2 l. more for every following one: What will the 10 horfes coft him? *Anf.* 110 l. Theorem VI. 7.

$$10 \times 11 = 110 \text{ l. } Anf.$$

PROB. VI.

Given the fum of the feries, the number of terms, and common difference, to find the extremes; that is, given V. III. IV. to find I. and II.

RULE.

Divide the fum of the feries by the number of terms, and the quot is half the fum of the extremes; then multiply the common difference into the number of terms minus unity, and the product divided by 2 quotes half the difference of the extremes. The half difference added to the half fum gives the greater extreme, and fubtracted leaves the leffer, by Theorem VII.

EXAMPLE.

A man received 300 l. at 12 payments, each payment exceeding the former by 4 l.: What was the firft, and what the laft payment?

12)300(25 half fum of the extremes.
$4 \times 11 = 44$, and 2)44(22 half difference of extremes.
And $25 + 22 = 47$ greateft extreme, or laft payment.
And $25 - 22 = 3$ leaft extreme, or firft payment.

MIXT QUESTIONS.

Queft. 1. A man travels 15 days, increafing every day's journey by 3 miles, his laft day's journey was 44 miles: What was his firft day's journey? and how many miles did he travel?

$3 \times 14 = 42$, and $44 - 42 = 2$, the miles he travelled the firſt day, by Prob. I.

And $2 + 44 = 46$, and $2)46(23$, and $23 \times 15 = 345$, the miles travelled in all, by Prob. V. *See* Theorem VI. 3.

Queſt. 2. A man clears a debt by payments in arithmetical proportion, the common difference being 5 l.; the firſt payment was 12 l. the laſt 57 l.: How many payments were there? and what the amount of the debt?

$57 - 12 = 45$, and $5)45(9$, and $9 + 1 = 10$, the number of payments, by Prob. III.

And $12 + 57 = 69$, and $69 \times 5 = 345$, the amount of the debt, by Prob. V. *See* Theorem VI. 2.

II. *Geometrical progreſſion.*

A geometrical progreſſion, or ſeries, is formed from ſome number aſſumed as the firſt term, by continual multiplication or diviſion; and is either increaſing or decreaſing.

Thus, 1 : 2 : 4 : 8 : 16 : 32 : 64, is a geometrical progreſſion or ſeries, formed from 1 aſſumed as the firſt term, and increaſing, being continually multiplied by 2; and may be continued upward to infinity.

And 243 : 81 : 27 : 9 : 3 : 1, is a geometrical progreſſion or ſeries, formed from 243 aſſumed as the firſt term, and decreaſing, being continually divided by 3, and may likewiſe be continued downward to infinity.

The multiplier or diviſor whereby the ſeries is continued upward or downward, is called the *common ratio.*

Note. The natural numbers, 1, 2, 3, 4, &c. are ſometimes ſet over a geometrical ſeries, to ſhew the diſtance of any term from unity, or from the firſt term; and in this caſe the natural numbers are called *indices,* or *exponents*; thus,

1.	2.	3.	4.	5.	6.	exponents.
3 :	6 :	12 :	24 :	48 :	96	ſeries.

But if the ſeries proceed from unity, it is uſual and convenient to place o over 1, thus,

0.	1.	2.	3.	4.	5.	6.	exponents.
1 :	2 :	4 :	8 :	16 :	32 :	64	ſeries.

In a geometrical progreſſion, or ſeries, five things occur to be conſidered; any three of which being given, the other two may be found. The five things are,

I. The leaſt term. } Extremes.
II. The greateſt term. }
III. The number of terms.
IV. The common ratio.
V. The ſum of all the terms.

The more uſeful properties of numbers in geometrical progreſſion are explained in the following theorems.

THEOREM I.

Any term of a geometrical ſeries is equal to the product of the leaſt term, multiplied continually into the common ratio, repeated as often as there are terms before or after it.

Thus in the increaſing ſeries, 3 : 6 : 12 : 24 : 48 : 96, whoſe common ratio is 2, the term 96 is equal to $3 \times 2 \times 2 \times 2 \times 2 \times 2$.

Again, in the decreaſing ſeries 162 : 54 : 18 : 6 : 2, whoſe ratio is 3, the term 162 is equal to $2 \times 3 \times 3 \times 3 \times 3$.

The reaſon is evident from the nature of the ſeries, and the manner of its formation.

In practice it will be convenient, in the firſt place, to multiply the common ratio continually into itſelf; that is, to raiſe it to a power whoſe index or exponent is equal to the number of terms minus unity, and then multiply this power into the leaſt term.

Thus, in the firſt of the above examples, $2 \times 2 \times 2 \times 2 \times 2 = 32$, and $32 \times 3 = 96$. Again, in the ſecond example, $3 \times 3 \times 3 \times 3 = 81$, and $81 \times 2 = 162$.

Hence, in any geometrical ſeries, that power of the common ratio whoſe index is equal to the number of terms minus unity, multiplied into the leaſt term, produces the greateſt; and the greateſt term divided by that power quotes the leaſt.

THEOREM II.

In a geometrical ſeries conſiſting of three, five, or any odd number of terms, the ſquare of the middle term is equal to the product of the extremes, or to the product of any two means equally diſtant from the ſaid middle term.

Thus, in the ſeries $1 : 2 : 4 : (8) : 16 : 32 : 64$, the ſquare of the middle term 8, viz. $8 \times 8 = 1 \times 64$, the extremes, or $= 2 \times 32$, or $= 4 \times 16$, the equidiſtant means.

The reaſon will appear by conſidering, that in four proportional numbers the product of the extremes is equal to the product of the means, as was demonſtrated in Multiplication of Vulgar Fractions; and when there are only three proportional numbers given, the middle term ſupplies the place of two, being the conſequent to the firſt, and the antecedent to the third term. Thus, $1 : 8 : 64$, expreſſed at more length is $1 : 8 :: 8 : 64$; and therefore $8 \times 8 = 1 \times 64$. Again, $2 : 8 : 32$, expreſſed more at large is, $2 : 8 :: 8 : 32$, and ſo $8 \times 8 = 2 \times 32$, &c.

COROLLARIES.

1. Hence a geometrical mean betwixt two given extremes is found by multiplying the extremes into one another, and extracting the ſquare root of the product. Thus the mean betwixt two and 32 is 8; for $2 \times 32 = 64$, and the ſquare root of 64 is 8. So $2 : 8 : 32$.

2. Hence alſo to an extreme and mean given, a third proportional, or the other extreme, is found by dividing the

the ſquare of the mean by the given extreme. Thus, to 4 : 8 given, the third proportional is 16; for $8 \times 8 = 64$, and 4)64(16, ſo 4 : 8 : 16. In like manner to 32 : 8 given, the third proportional is 2; for $8 \times 8 = 64$, and 32)64(2; ſo 32 : 8 : 2.

3. Hence likewiſe, in a ſeries not proceeding from unity, the ſquare of any term divided by the firſt term quotes a term of a double exponent minus unity. Thus in the 1 ſeries following, $12 \times 12 = 144$, and 3)144(48, the fifth term; and in the 2 ſeries, 48)144(3, the fifth term.

	1.	2.	3.	4.	5.
1 ſeries	3 :	6 :	12 :	24 :	48.
2 ſeries	48 :	24 :	12 :	6 :	3.

THEOREM III.

In a geometrical ſeries conſiſting of four, ſix, or any even number of terms, the product of the extremes is equal to the product of the two middle terms, or to the product of any two means equally diſtant from the extremes.

Thus in the ſeries, 2 : 4 : (8 : 16) : 32 : 64, the product of $2 \times 64 = 8 \times 16$, or $= 4 \times 32$.

The reaſon is plain: for in four proportional numbers the product of the extremes is equal to the product of the means; and any pair of equidiſtant means may be eſteemed middle terms.

COROLLARIES.

1. Hence to an extreme and two means given, the other extreme, or fourth proportional, is found by dividing the product of the means by the given extreme. Thus to 4 : 8 : 16, the fourth proportional is 32; for $8 \times 16 = 128$, and 4)128(32; ſo 4 : 8 : 16 : 32. And this takes place though the proportion be disjunct. Thus to 48 : 24 :: 6, the fourth proportional is 3; for $24 \times 6 = 144$, and 48)144(3; ſo 48 : 24 :: 6 : 3.

2.

2. Hence likewise, when two extremes and a mean are given, the other mean is found by dividing the product of the two extremes by the given mean. Thus, to 2 : 4 : 16, the other mean is 8 ; for $2 \times 16 = 32$; and 4)32(8 ; so 2 : 4 : 8 : 16. This likewise takes place tho' the proportion be disjunct. Thus, to 2 :: 32 : 64, the other mean is 4 ; for $2 \times 64 = 128$, and 32)128(4 ; so 2 : 4 :: 32 : 64.

3. From this and the preceding problem, it follows, that in a series proceeding from unity, any term squared produces a term of a double exponent ; and any two or more terms multiplied into one another, produce a term whose exponent is the sum of all their exponents. Thus, in the following series, $8 \times 8 = 64$, whose exponent is $6 = 2 \times 3$; and $4 \times 8 = 32$, whose exponent is $5 = 2 + 3$; and $2 \times 4 \times 8 = 64$, whose exponent is $6 = 1 + 2 + 3$.

0.	1.	2.	3.	4.	5.	6.
1 :	2 :	4 :	8 :	16 :	32 :	64

THEOREM IV.

In a geometrical series, the quot of the greatest extreme divided by the least, is equal to that power of the common ratio whose exponent is the number of terms minus unity.

Thus, in the series, 3 : 6 : 12 : 24 : 48 : 96, the quot of $3)96(32 = 2 \times 2 \times 2 \times 2 \times 2 = 32$, the fifth power of 2, the common ratio of the series.

The reason is obvious : for, by Theorem I. the greatest term divided by such a power of the ratio, quotes the first term ; and any dividend divided by the quot, gives the divisor.

COROLLARIES.

1. Hence the extremes and number of terms being given, the common ratio may be found ; viz. divide the greatest extreme by the least, the quot is a power whose

whose exponent is the number of terms minus unity, and the root extracted is the common ratio.

2. Hence likewise we have a method of finding several mean proportionals betwixt two given numbers, viz. divide the greater by the lesser, esteem the quot a power whose index is greater by unity than the number of means proposed, and the root of this power extracted is the ratio; by which multiply the lesser of the given numbers continually, and the several products are the means required.

THEOREM V.

In a geometrical series, the difference of the extremes divided by the common ratio minus unity, quotes the sum of all the terms, except the greatest.

Thus, in the series, $3 : 9 : 27 : 81 : 243$, the greatest term $243 - 3 = 240$, and the ratio $3 - 1 = 2)240(120 = 3 + 9 + 27 + 81$.

The truth of the proposition will be evident from the following considerations, viz. In a series whose ratio is 2, any term minus the least is equal to the sum of all the lesser terms. Thus, in the series, $1 : 2 : 4 : 8 : 16$, the second term $2 - 1 = 1$; and $4 - 1 = 1 + 2$; and $8 - 1 = 1 + 2 + 4$; and $16 - 1 = 1 + 2 + 4 + 8$. If the ratio be 3, any term minus the least is double to the sum of all the lesser terms. Thus, in the series, $18 : 6 : 2$, the second term $\frac{6 - 2}{2} = 2$, and $\frac{18 - 2}{2} = 6 + 2$. If the ratio be 4, any term minus the least is triple of all the lesser terms, &c. And therefore, universally, the difference of the extremes divided by the common ratio minus unity, quotes the sum of all the terms, except the greatest.

COROLLARIES.

1. The least term multiplied into that power of the ratio whose index is the number of terms, gives the next higher term of the series continued; from which, therefore, if the least term be subtracted, the remainder divided

vided by the ratio minus unity, will quote the ſum of the given ſeries.

2. If a decreaſing ſeries be ſuppoſed infinite, the leaſt extreme vaniſhes, or becomes 0; and in this caſe, if the greateſt extreme be multiplied into the common ratio, the product divided by the ratio minus unity, will quote the ſum of the ſeries. Thus, $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$, &c. $= \frac{1 \times 2}{2 - 1} = 2$; and $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}$, &c. $= \frac{1 \times 3}{3 - 1} = \frac{3}{2} = 1\frac{1}{2}$.

3. The difference of the extremes of a ſeries divided by the difference of the ſum and greater extreme, quotes the ratio minus unity.

We ſhall now ſubjoin a few problems, whoſe ſolutions flow directly from the above theorems, or their corollaries.

PROB. I.

Given the greateſt term, the number of terms, and common ratio, to find the leaſt term; that is, Given II. III. IV. to find I.

RULE.

Raiſe the common ratio to a power whoſe index is the number of terms minus unity; by this divide the greateſt term, and the quot is the leaſt term, by Theorem I.

EXAMPLE.

A farmer buys ſix cows, whoſe prices were in geometrical progreſſion, the common ratio being 2, the price of the laſt was 96 crowns: What was the price of the firſt? *Anſ.* 3 crowns.

1, 2, 3, 4, 5
$2 \times 2 \times 2 \times 2 \times 2 = 32$, and 32)96(3 crowns. *Anſ.*

PROB.

PROB. II.

Given the leaſt term, the number of terms, and common ratio, to find the greateſt term; that is, Given I. III. IV. to find II.

RULE.

Raiſe the common ratio to a power whoſe index is the number of terms minus unity, multiply this by the leaſt term, and the product is the greateſt, by Theorem I.

EXAMPLE.

A nobleman had nine ſons, to whom he left his eſtate, divided into portions in geometrical progreſſion, the common ratio being 3; the youngeſt ſon got the leaſt portion, being only 50 l.: What did the eldeſt ſon get? *Anſ.* 328050 l.

1. 2. 3. 4. 5. 6. 7. 8.
$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 6561$, and
$6561 \times 50 = 328050$ l. *Anſ.*

PROB. III.

The firſt term and common ratio of a ſeries not proceeding from unity being given, to find any remote term, without producing all the intermediate terms.

RULE.

Find a few of the terms, by multiplying or dividing the firſt term continually by the common ratio, and over the terms thus found place their exponents; ſquare the greateſt of the found terms; which ſquare, divided by the firſt term, will quote a term of a double exponent minus unity. Again, the ſquare of the term laſt found, divided by the firſt, will quote another term of a double exponent minus unity. Thus proceed, till you either find the term ſought, or one near it; and from a near term, the one ſought may be found by means of the ratio. Theorem II. cor. 3.

EXAMPLE.

A ſum of money was divided among ten perſons; their ſhares in geometrical progreſſion, and the common ratio, 3: the firſt perſon got 4 l.: What was the tenth perſon's ſhare? *Anſ.* 78732 l.

$$\begin{array}{l} 1. \quad 2. \quad 3. \qquad\quad 5. \qquad\qquad 9. \qquad\quad 10. \\ 4 : 12 : 36 \text{——} 324 \text{——} 26244 : 78732.\ \textit{Anſ.} \end{array}$$

$36 \times 36 = 1296 \div 4 = 324$ the fifth term.

$324 \times 324 = 104976 \div 4 = 26244$, the ninth term.

The tenth term is found by multiplying the ninth into the common ratio 3.

PROB. IV.

The common ratio of a ſeries proceeding from unity being given, to find any remote term, without producing all the intermediate terms.

RULE.

Find ſome few terms by means of the ratio, over which place their exponents; then obſerve what exponents added will make up the exponent of the term ſought, or of ſome term near it; and the terms ſtanding under the ſaid exponents, multiplied into one another, will produce the term anſwering to the ſum of the exponents. Theorem III. cor. 3.

Note. In a ſeries proceeding from unity, as 0 ſtands over 1, the exponent of every term will be one ſhort of the number of terms from the beginning of the ſeries. Thus, the exponent of the fifth term is 4, of the ſixth term 5, &c.

EXAMPLE.

A graſier bought 14 ſheep, at a farthing for the firſt, a halfpenny for the ſecond, &c. ſtill doubling the price for every ſubſequent one; and was to pay only the price of the laſt ſheep for the whole: What ſum had he to pay? *Anſ.* L. 8 : 10 : 8.

O.

0. 1. 2. 3. 4. 5.
1 : 2 : 4 : 8 : 16 : 32

$1 + 3 + 4 + 5 = \quad 13$ L. s. d.
$2 \times 8 \times 16 \times 32 = 8192 = $ 8 10 8 *Anſ.*

Or thus,

$3 + 5 + 5 = \quad 13$
$8 \times 32 \times 32 = 8192$

Or thus,

$1 + 3 + 4 + 5 = \quad 13$
$2 \times 8 \times 16 \times 32 = 8192$

P R O B. V.

Given the extremes and common ratio, to find the number of terms; that is, Given I. II. IV. to find III.

R U L E.

Divide the greateſt extreme by the leaſt, raiſe the common ratio to a power equal to the quot, and the index of that power plus 1 is the number of terms. Theorem IV.

E X A M P L E.

A gentleman purchaſed ſome acres of ground, whoſe prices were in geometrical progreſſion, the common ratio being 3; the price of the firſt acre was 3d. and of the laſt 59049 d.: How many acres did he purchaſe? *Anſ.* 10 acres.

3)59049(19683

1. 2. 3. 4. 5. 6. 7. 8. 9. and $9 + 1 = 10$ *Anſ.*
And $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 19683$

P R O B. VI.

Given the extremes and common ratio, to find the ſum of the ſeries; that is, Given I. II. IV. to find V.

R U L E.

Divide the difference of the extremes by the ratio mi-

 nus

nus unity, and the quot added to the greatest extreme, gives the sum of the series. Theorem V.

EXAMPLE.

A gentleman who had a daughter married on New-year's day, gave the husband towards her portion 4 shillings, promising to triple that sum the first day of every month, for nine months after the marriage; the sum paid on the first day of the ninth month was 26244 shillings: What was the Lady's portion? *Ans.* 39364 shillings, or 1968 l. 4 s.

26244 — 4 = 26240, and 3 — 1 = 2)26240(13120, and 13120 + 26244 = 39364 s. = 1968 l. 4 s.

PROB. VII.

Given the least extreme, the number of terms, and common ratio, to find the sum of the series; that is, Given I. III. IV. to find V.

RULE.

Find, by Prob. III. or IV. the term next after the greatest extreme; from this term subtract the least term; and the remainder, divided by the ratio minus unity, will quote the sum of the series. Theorem V. cor. 1.

EXAMPLE I.

A corn-merchant buys 12 stacks of wheat, and was to pay 2 d. for the first stack, 6 d. for the next, tripling the price for every following stack: What sum had he to pay? *Ans.* 531440 d. or L. 2214 : 6 : 8.

1. 2. 3. 4. —— 7. —— 13.
2 : 6 : 18 : 54 —— 1458 —— 1062882
54 × 54 = 2916 ÷ 2 = 1458
1458 × 1458 = 2125764 ÷ 2 = 1062882
1062882 — 2 = 1062880, and 3 — 1 =
2)1062880(531440 d. = L. 2214 : 6 : 8.

EX-

EXAMPLE II.

A gentleman buys a fine houſe, in which were 24 threſholds; and, in name of price, was to lay a farthing on the firſt threſhold, a halfpenny on the ſecond, a penny on the third, doubling the ſum on every following threſhold: what would the houſe coſt him? *Anſ.* 16777215 farthings, or L. 17476 : 5 : $3\frac{3}{4}$.

$$
\begin{array}{l}
0.\ 1.\ 2.\ 3.\ 4.\ 5.\ \quad 9.\ \quad 24. \\
1 : 2 : 4 : 8 : 16 : 32 \text{ —— } 512 \text{ —— } 16777216 \\
4 + 5 = 9 \\
16 \times 32 = 512 \\
9 + 9 + 2 + 4 = 24 \\
512 \times 512 \times 4 \times 16 = 16777216 \quad \text{L.} \quad \text{s.} \quad \text{d.} \\
16777216 - 1 = 16777215 \text{ f.} = 17476 \quad 5 \quad 3\frac{3}{4}
\end{array}
$$

The ratio minus unity being 1, there is no occaſion to divide by it.

EXAMPLE III.

What will a horſe coſt by tripling the 32 nails in his ſhoes with a farthing? *Anſ.* 926510094425920 farthings, or 965114682527 l.

EXAMPLE IV.

What will 40 drove of cattle coſt, by tripling each drove with a farthing? *Anſ.* 6332117426592150 l. 8 s. 4 d.

PROB. VIII.

Given the extremes, and ſum of the ſeries, to find the common ratio, and number of terms; that is, Given I. II. V. to find IV. III.

RULE.

Divide the difference of the extremes by the difference of the ſum and greater extreme, and the quot is the ratio minus unity, by Theorem V. cor. 3. The ratio

ratio being thus obtained, find the number of terms by Prob. V.

EXAMPLE.

A ſold his eſtate for 21844 l. which was paid by ſeveral payments, in geometrical progreſſion; the firſt was 4 l. the laſt 16384 l.: What was the ratio, and how many payments? *Anſ.* The ratio 4, payments 7.

```
21844    16384
16384        4
-----    -----
 5460)   16380(3 + 1 = 4 the common ratio.
       4)16384(4096
```

1. 2. 3. 4. 5. 6. and 6 + 1 = 7 the number of payments.
4 × 4 × 4 × 4 × 4 × 4 = 4096.

CHAP. XII.

INTEREST.

Intereſt is a ſmall ſum of money paid by the borrower to the lender for the uſe of a greater ſum, at a certain rate *per cent. per annum*; and is either ſimple or compound.

I. *Simple Intereſt.*

Simple intereſt is that which ariſes purely or only from the principal, or ſum lent.

The ſum of principal and intereſt is called the *amount*. The rate by the laws of Britain, ever ſince 1714, cannot exceed 5 *per cent.* but may be leſs.

The year is ſuppoſed always to conſiſt of 365 days, and the 29th of February in leap years is not reckoned: for no more intereſt can be legally charged for leap than for a common year.

The day computed from is not reckoned, but the day computed to is. Few chuſe to compute by months,

a month being no aliquot part of a year. The computation by years and quarters is uſual, but years and days ſtill more ſo.

The operations may be rendered more ſimple by aſſuming the intereſt of 1 l. for a year, as the rate. Thus, at 4, $4\frac{1}{4}$, $4\frac{1}{2}$, $4\frac{3}{4}$, and 5 *per cent.* the intereſt or rate for 1 l. is .04, .0425, .045, .0475, .05, found by ſaying, As 100 : 4 : : 1 : .04, &c.

The principal and every year's amount make a ſeries of arithmetical proportionals, the rate being the common difference; as under.

		1	2	3	4	5 years.	
Prin.	1 :	1.05 :	1.10 :	1.15 :	1.20 :	1.25	.05 diff.
Prin.	100 :	105 :	110 :	115 :	120 :	125	5 diff.

PROB. I.

Principal, rate, and time, in years, given, to find the intereſt, and conſequently the amount.

Multiply the principal, the rate of 1 l. and the time, continually, and the laſt product is the intereſt.

Or,

Multiply the principal, the rate of 100 l. and the time, continually, and the laſt product, divided by 100, quotes the intereſt.

EXAMPLE I.

What is the intereſt of 584 l. 6 s. 8 d. for $3\frac{3}{4}$ years, at $4\frac{1}{2}$ *per cent.*?

	$584.\dot{3}$		
	$.045$		
	$2921\dot{6}$		
	$23373\dot{3}$		
	26.2950		
	$3\frac{3}{4}$		
4)	78.885		
	19.72125	L. s. d.	
Int.	98.60625	= 98 12 $1\frac{1}{2}$	
	Principal	584 6 8	
	Amount	682 18 $9\frac{1}{2}$	

Or thus:

	$584.\dot{3}$
	$4\frac{1}{2}$
	$2337.3\dot{3}$
	$292.1\dot{6}$
	2629.50
	3.75
	131475
	184065
	78885
100)	98.60625 interest.

The amount is found by adding the principal to the interest as above: but the amount may also be found thus: To the product of the rate and time add 1, and multiply by the principal.

Time	3.75	1.16875	
Rate	$.045$	$584.\dot{3}$	
	1875	467500	
	1500	935000	
Product	$.16875$	584375	
		682.55000	
		$.38958\dot{3} = \frac{1}{3}$	L. s. d.
	Amount	$682.93958\dot{3}$	= 682 18 $9\frac{1}{2}$

The interest may be found by subtracting the principal from the amount.

If the given time be days, or years and days, reduce the days to the decimal of a year, and then work as before. Or, reduce the years to days, then work, and divide the result by 365.

The reason both of this rule, and of those assigned in the subsequent problems, may be deduced from the compound proportion following.

Prin.

Prin.	*Ye.*	*Int.*		*Prin.*	*Year.*	*Int.*
100 × 1 : $4\frac{1}{2}$			::	584 6 8 × $3\frac{3}{4}$	:	98 12 $1\frac{1}{2}$
Or, 1 × 1 : .045			::	584.3 × 3.75	:	98.60625

But intereſt is more uſually computed by ſome practical method: and theſe methods are numerous, and various in their own nature; and differ alſo as the rate varies, and as the given time is years or days.

I. *The given time years.*

RULES.

1. When the rate is 5 *per cent.* and the time 1 year, $\frac{1}{20}$ of the principal is the intereſt ſought, becauſe the rate 5 is $\frac{1}{20}$ of 100.

This is the moſt ſimple caſe that can happen; and becauſe 5 *per cent.* is 1 s. per l. you may readily compute the intereſt by eſteeming the pounds ſo many ſhillings, and the ſhillings ſo many half pence, adding $\frac{1}{5}$, and if there be any pence, every 6 d. is $1\frac{1}{5}$ farthing. Thus, the intereſt of 54 l. 16 s. for 1 year, at 5 *per cent.* is 54 s. $19\frac{1}{5}$ half pence, or 2 l. 14 s. 9 d.

2. When the given time is any number of years, for 2 years take $\frac{2}{20}=\frac{1}{10}$; for 3 years take $\frac{1}{20}$ thrice; for 4 take $\frac{4}{20}=\frac{1}{5}$; for 5 take $\frac{5}{20}=\frac{1}{4}$; for 10 take $\frac{10}{20}=\frac{1}{2}$, &c.; or find the intereſt for 1 year, and multiply the intereſt ſo found by the years in the queſtion; as in Ex. 2.

3. When the rate is any other than 5 *per cent.* firſt work for 5 *per cent.* and then to or from the intereſt thus found, add or ſubtract the correſpondent part; or multiply $\frac{1}{5}$ of the intereſt at 5 *per cent.* by the given rate; as in Ex. 3.

4. When the given time is years and quarters, or years and aliquot parts, find the intereſt at 5 *per cent.* for the years as taught above; for the quarters or parts take aliquot parts; and then multiply $\frac{1}{5}$ of the intereſt thus found by the given rate. Or, multiply the rate into the time, eſteem the product a new rate, to which compute the intereſt; as in Ex. 4. 5. and 6.

EXAMPLE II.

What is the intereſt of 684 l. 16 s. 8 d. for 15 years, at 5 *per cent.*?

Years.	*L.*	*s.*	*d.*
	684	16	8
$10 = \frac{1}{2}$	342	8	4
$5 = \frac{1}{2}$	171	4	2
Anſ.	513	12	6

Or thus,

20)	684	16	8
	34	4	10
$15 = 5 \times 3$			5
	171	4	2
			3
Anſ.	513	12	6

EXAMPLE III.

What is the intereſt of 438 l. 14 s. 6 d. for 12 years, at $4\frac{1}{2}$ *per cent.*?

Y.	*L.*	*s.*	*d.*
	438	14	6
$10 = \frac{1}{2}$	219	7	3
$2 = \frac{1}{10}$	43	17	5.4
at 5 *p. c.*	263	4	8.4
Deduce $\frac{1}{10}$	26	6	5.64
Anſ.	236	18	2.76

Or thus,

20)	438	14	6
	21	18	8.7
			12
5)	263	4	8.4
	52	12	11.28
			$4\frac{1}{2}$
	210	11	9.12
	26	6	5.64
Anſ.	236	18	2.76

EXAMPLE IV.

What is the intereſt of 317 l. 16 s. for $5\frac{3}{4}$ years, at $3\frac{1}{2}$ *per cent.*?

L.

Y.	L.	s.	
	317	16	
$5 = \frac{1}{4}$	79	9	
$\frac{1}{2} = \frac{1}{10}$	7	18	10.8
$\frac{1}{4} = \frac{1}{2}$	3	19	5.4
	5)91	7	4.2
	18	5	5.64
			$3\frac{1}{2}$
	54	16	4.92
	9	2	8.82
Anſ.	63	19	1.74

Or thus,

$5\frac{3}{4} \times 3\frac{1}{2} = 20\frac{1}{8}$

p. c.			
	317	16	
$10 = \frac{1}{10}$	31	15	7.2
$5 = \frac{1}{2}$	15	17	9.6
$5 = \frac{1}{2}$	15	17	9.6
$\frac{1}{8} = \frac{1}{40}$		7	11.34
Anſ.	63	19	1.74

EXAMPLE V.

What is the intereſt of 79 l. 16 s. for $5\frac{1}{2}$ years, at $4\frac{3}{8}$ *per cent.*?

Y.	L.	s.	
	79	16	
$5 = \frac{1}{4}$	19	19	
$\frac{1}{2} = \frac{1}{10}$	1	19	10.8
	5)21	18	10.8
	4	7	9.36
			$4\frac{3}{8}$
	17	11	1.44
$\frac{2}{8} = \frac{1}{4}$	1	1	11.34
$\frac{1}{8} = \frac{1}{2}$		10	11.67
Anſ.	19	4	0.45

Or thus,

$5\frac{1}{2} \times 4\frac{3}{8} = 24\frac{1}{16}$

p. c.			
	79	16	
$20 = \frac{1}{5}$	15	19	2.4
$4 = \frac{1}{6}$	3	3	10.08
$\frac{1}{16} = \frac{1}{64}$			11.97
Anſ.	19	4	0.45

EXAMPLE VI.

What is the intereſt of 648 l. 8 s. for $6\frac{3}{4}$ years, at $4\frac{1}{4}$ *per cent.*?

Y.		L.
		648.4
5	$= \frac{1}{4}$	162.1
1	$= \frac{1}{5}$	32.42
$\frac{1}{2}$	$= \frac{1}{2}$	16.21
$\frac{1}{4}$	$= \frac{1}{2}$	8.105

5)218.835

43.767
$4\frac{1}{4}$

175.068
10.94175

	L.	s.	d.
Anſ. 186.00975 =	186	0	$2\frac{1}{4}$

Or thus,

$6\frac{3}{4} \times 4\frac{1}{4} = 28\frac{11}{16}$

p. c.		648.4
20	$= \frac{1}{5}$	129.68
4	$= \frac{1}{5}$	25.936
4	$= \frac{1}{5}$	25.936
$\frac{11}{16}$	$= \frac{11}{64}$	4.45775

Anſ. 186.00975

II. *The given time days.*

R U L E S.

1. Multiply the principal by the number of days, and the product divided by 7300 quotes the intereſt at 5 *per cent.*: and if the rate be any other than 5 *per cent.* adjuſt the matter by aliquot parts, as in Ex. 7.

2. Divide any principal by 73, and the quot will be the intereſt for 100 days at 5 *per cent.* and for the remaining days take aliquot parts, as in Ex. 8.

3. If the number of the given days be 73, or any multiple of 73, ſuch as, 146, 219, 292, 365, &c. divide the principal by 100, and the quot will be the intereſt for 73 days at 5 *per cent.*; which multiplied by the number denoting the multiple, will produce the intereſt for the given days, as in Ex. 9.

4. If the principal be 100 l. eſteem the days ſo many pounds principal, divide by 73, and the quot will be the intereſt of 100 l. for the given number of days, at 5 *per cent.* as in Ex. 10.

The reaſon of the above rules will appear from the following

following compound proportion, in which $p.$ is put to denote the principal, and $d.$ the number of days.

$$\begin{array}{c} \textit{Prin. Days. Int.} \\ 100 \times 365 : 5 :: p. \times d. \end{array}$$

That is, $\dfrac{5 \times p. \times d.}{100 \times 365} = \dfrac{p. \times d.}{100 \times 73} = \dfrac{p. \times d.}{7300}$, Hence Rule 1.

$$\begin{array}{c} \textit{Prin. Days. Int. Days.} \\ \text{Again, } 100 \times 365 : 5 :: p. \times 100 \end{array}$$

That is, $\dfrac{5 \times p. \times 100}{100 \times 365} = \dfrac{5 \times p.}{365} = \dfrac{p.}{73}$, Hence Rule 2.

The truth of Rule 3. is evident; for if 73 dividing any principal quotes the intereſt for 100 days, it follows, that 100 dividing any principal will quote the intereſt for 73 days.

The reaſon of Rule 4. is alſo obvious; for the intereſt of 100 l. for 50 days will be equal to the intereſt of 50 l. for 100 days.

EXAMPLE VII.

What is the intereſt of 245 l. 13 s. 4 d. from the 21ſt of March to the 22d of November, at $4\frac{3}{4}$ *per cent.*?

Days,

	Days.
Mar.	10
Apr.	30
May	31
June	30
July	31
Aug.	31
Sept.	30
Oct.	31
Nov.	22
	246

$$
\begin{array}{r}
L. \\
245.\cancel{0} \\
246 \\
\hline
14740 \\
982\cancel{0}6 \\
491\cancel{3}33 \\
\hline
\end{array}
$$

20)

73|00)60434.0(8.2786, at 5 *per cent.*

584 .. .   .4130, at $\frac{1}{4}$ *per cent.*

$$
\begin{array}{r}
\hline
203 \\
146 \\
\hline
574 \\
511 \\
\hline
630 \\
584 \\
\hline
460 \\
438 \\
\hline
(22)
\end{array}
$$

7.8647 = L. 7 17 $3\frac{1}{2}$, at $4\frac{3}{4}$ *per cent.* *Ans.*

EXAMPLE VIII.

What is the intereſt of 378 l. 3 s. $8\frac{1}{2}$ d. for 275 days, at $4\frac{1}{2}$ *per cent.* ?

L.

L.	s.	d.	L.	s.	d.	Days.
73)378	3	$8\frac{1}{2}$	(5	3	7.349,	for 100
365					2	
13			10	7	2.698,	for 200
20			2	11	9.674,	for 50 = $\frac{1}{2}$
263			1	0	8.669,	for 20 = $\frac{1}{2}$
219				5	2.167,	for 5 = $\frac{1}{4}$
44			10)14	4	11.208,	at 5 *per cent.*
12			1	8	5.920,	at $\frac{1}{2}$ *per cent.*
536.5			*Anſ.* 12	16	5.288,	at $4\frac{1}{2}$ *per cent.*
511						
255			Or, L. 12	16	$5\frac{1}{4}$	
219						
360						
292						
680						
657						
(23)						

EXAMPLE IX.

What is the intereſt of 864 l. 19 s. 8 d. for 219 days, at $4\frac{5}{8}$ *per cent.*?

L.	s.	d.	L.	s.	d.	Days.
100)8|64	19	8	(8	12	11.96,	for 73
20					3	
12|99			25	18	11.88,	for 219, at 5 *per cent.*
12			1	18	11.09,	at $\frac{3}{8}$ *per cent.* = $\frac{3}{40}$
11|96			24	0	0.79,	at $4\frac{5}{8}$ *per cent.* *Anſ.*

EXAMPLE X.

What is the intereſt of 100 l. for 254 days, at $4\frac{7}{8}$ *per cent.*?

L.

	L.	s.	d.	
73) 254 (	3	9	7.068,	at 5 *per cent.*
219		1	8.876,	at $\frac{1}{8}$ *per cent.* = $\frac{1}{40}$
35	3	7	10.192,	at $4\frac{7}{8}$ *p. c.* for 254 days. *Anſ.*

35
20
700
657
43
12
516
511
500
438
620
584
(36)

If the given principal be any multiple of 100, work for 100 as above, and then multiply the reſult by the number denoting the multiple.

The computing of intereſt occurs ſo frequently in practice, that men of buſineſs, for the ſake of eaſe and diſpatch, generally uſe tables conſtructed for that purpoſe. The following tables give the intereſt at 5 *per cent.*; whence the intereſt at any other rate may eaſily be found.

The intereſt of one pound, at 5 *per cent.*

	TABLE I. per year.	TABLE II. per day.	TABLE III. per quarter.	TABLE IV. per month.
1	.05	.0001369863	.0125	$.0041\dot{6}$
2	.1	.0002739726	.025	$.008\dot{3}$
3	.15	.0004109589	.0375	.0125
4	.2	.0005479452	.05	$.01\dot{6}$
5	.25	.0006849315	.0625	$.0208\dot{3}$
6	.3	.0008219178	.075	.025
7	.35	.0009589041	.0875	$.0291\dot{6}$
8	.4	.0010958904	.1	$.0\dot{3}$
9	.45	.0012328767	.1125	.0375

The

The above tables are conſtructed by the rules already aſſigned for finding the intereſt of any principal for any given time. The quarter in Table III. is $\frac{1}{4}$ of a year, or 91 days 6 hours; and the month in Table IV. is $\frac{1}{12}$ of a year, or 30 days 10 hours. In uſing theſe tables obſerve the following

RULES.

1. Reſolve the given number of years or days into its conſtituent parts.

2. Seek the ſignificant figure of each conſtituent part on the left; oppoſite to which, under per year, or per day, &c. according to the denomination of the given time, you have the decimal to be taken out.

3. Move the decimal point in each decimal ſo many places to the right as there are ciphers in the conſtituent part; and, in taking out for decimal figures, move the decimal point ſo many places to the left as the decimal figure is to the right.

4. Add the decimals thus collected from the tables, and their ſum is the intereſt ſought.

5. If the given time be of different denominations, ſuch as years and days, find the intereſt for the years and days ſeparately, and the ſum of the reſults will be the intereſt ſought. Or, reduce the years to days, and then collect from Table II. Do the like when the time is given in years and quarters, or years and months, &c.

6. If the given rate be not 5 *per cent.* firſt find the intereſt at 5 *per cent.* and then multiply one fifth of the intereſt ſo found by the given rate.

The decimals in the tables are complete, and conſequently will, when all the figures are uſed, give a complete anſwer; but yet, in moſt caſes, four or five figures will be ſufficiently accurate.

EXAMPLE I.

What is the intereſt of 3485 l. for 1 year, at $4\frac{3}{4}$ *per cent.*?

In Table I. opposite to you find — { 3000 — 150; 400 — 20; 80 — 4; 5 — .25

```
            5)174 25  at 5 per cent.
              34.8    at 1 per cent.
                 4¾
             139.40
              17.425
               8.7125          L.   s.  d.
Anſ.  165.5375 = 165  10  9
```

EXAMPLE II.

What is the intereſt of 3485 l. for 1 day, at $4\frac{3}{4}$ *per cent.?*

In Table II. oppoſite to you find — { 3000 — .41095; 400 — .05479; 80 — .01095; 5 — .00068

```
            5).47737  at 5 per cent.
              .09547  at 1 per cent.
                  4¾
              .38188
              .04773
              .02386       s.  d.
              .45347  =  9   0¾  Anſ.
```

EXAMPLE III.

What is the intereſt of 284 l. 5 s. for 15 years, at 5 *per cent.?*

Multiply the principal 284.25
by the number of years 15

Reſolve the product 4263.75 into conſtituent parts, and then proceed as before.

4000

4000	——	200
200	——	10
60	——	3
3	——	.15
.7	——	.035
.05	——	.0025

L. s. d.
213.1875 = 213 3 9 Anf.

EXAMPLE IV.

What is the intereft of 564 l. for 238 days, at $4\frac{1}{2}$ *per cent.*?

Multiply the principal 564
by the number of days 238

Refolve the product 134232 into conftituent parts, and then proceed as formerly.

100000	——	13.6986
30000	——	4.1095
4000	——	.5479
200	——	.0273
30	——	.0041
2	——	.0002

5)18.3876 at 5 *per cent.*

3.6775 at 1 *per cent.*
$4\frac{1}{2}$

14.7100
1.8387

L. s. d.
16.5487 = 16 10 $11\frac{1}{2}$ *Anf.*

EXAMPLE V.

What is the intereft of 78 l. for $4\frac{1}{4}$ years, at $4\frac{1}{8}$ *per cent.*?

$$78 \times 4.25 = 331.5$$

By Table I.

300 ——	15
30 ——	1.5
1 ——	.05
.5 ——	.025

5)16.575 at 5 *per cent.*

3.315 at 1 *per cent.*

$4\frac{1}{8}$

13.260

$.414 = \frac{1}{8}$

		L.	s.	d.
Anſ. 13.674	=	13	13	$5\frac{3}{4}$

Or, 78×17 quarters $= 1326$

By Table III.

1000 ——	12.5
300 ——	3.75
20 ——	.25
6 ——	.075

5)16.575 at 5 *per cent.*

3.315 at 1 *per cent.*

$4\frac{1}{8}$

13.260

$.414 = \frac{1}{8}$.

Anſ. 13.674 as before.

EXAMPLE VI.

What is the intereſt of 85 l. 14 s. 6 d. for 3 years 11 months, at $4\frac{5}{8}$ *per cent.*?

85.725

85.725 × 47 months = 4029.075

By Table IV.

4000	——	16.6666
20	——	.0833
9	——	.0375
.07	——	.0002
.005	——	.0000

5)16.7876 at 5 *per cent.*

3.3575 at 1 *per cent.*
$4\frac{5}{8}$

13.4300
1.6787 = $\frac{4}{8}$
.4196 = $\frac{1}{8}$

		L.	s.	d.
Anſ. 15.5283	=	15	10	$6\frac{3}{4}$

In calculating intereſt on caſh-accounts, or accounts-current, where partial payments are made, and the account cleared within twelve months, multiply the principal and the ſeveral balances into the number of days they are at intereſt, and the ſum of theſe products divided by 7300, will quote the intereſt at 5 *per cent.*; and for any other rate multiply one fifth of the intereſt thus found by the given rate, and the product will be the intereſt ſought; as in the following examples.

EXAMPLE I.

Lent A B, the 10th of June 1765, the ſum of 800 l. Sterling, and received the ſame back in partial payments, as follows: What intereſt is due at 5 *per cent.*?

1765.			L.	s.	d.	Da.	Products.
June	10	Principal lent -	800			60	48000
Aug.	9	Received -	200	10			
		New principal	599	10		67	40166.5
Octob.	15	Received -	328	15	6		
		New principal	270	14	6	38	10287.55
Nov.	22	Received -	170	14	6		
		New principal	100			31	3100
Dec.	25	Recd in full of principal	100				
							101554.05

$7300 = 73 \times 100$

$73)1015.5405(13.9115 = 13$ l. 18 s. $2\frac{3}{4}$ d. *Ans.*

Banks, or bankers, ſometimes borrow at 4 *per cent.* but lend at 5 *per cent.*; and the perſon to whom they give a caſh-credit, has no occcaſion to keep money by him, but gives it into the bank, and receives 4 *per cent.* intereſt for the balance of caſh due to him; but when the balance runs againſt him, he pays intereſt at the rate of 5 *per cent.* In this caſe it will be proper to conſider the money lent or paid by the bank as Dr, and the money received by the bank as Cr, and to make two columns for products, viz. one for the Dr products, and the other for the Cr products. The intereſt ariſing from the Dr products is to be computed at 5 *per cent.* and that from the Cr products at 4 *per cent.* and the difference of theſe two is the balance of intereſt due by the perſon to the bank, or by the bank to him.

EXAMPLE II.

Given A B a caſh-credit at 5 *per cent.* for the balances due by him, allowing him 4 *per cent.* for ſuch balances as may be due to him: What intereſt will be due, and by whom, in conſequence of the following tranſactions?

1765.

1765.		L. s. d.	Da.	Dr	Cr
Jan. 8.	Dr	845 6 8	102	86224	
April 20.	Cr	1265 6 8			
	Cr	420	86		36120
July 15.	Dr	968 10 0			
	Dr	548 10 0	77	42234.5	
Sept. 30.	Cr	848 10 0			
	Cr	300	32		9600
Nov. 1.	Dr	500			
	Dr	200	44	8800	
Dec. 15.	Cr	200			
				137258.5	45720
		At 5 *p. c.* = L.		18 15 6	= L. 5, at 4 *p. c.*
				5 00 0	
				13 15 6	int. due by A B.

If the rate of intereſt on both ſides be the ſame, you have only to divide the difference of the ſums of the Dr and Cr products by 7300.

When partial payments are made on bonds or bills at any interval greater than a year, the legal and uſual method is, to add the intereſt at the times of payment to the principal, from that amount deducting the payment. Bankers avoid ſuch dilatory payments by taking care to have all their caſh-accounts ſettled within the year, this being the ſhorteſt or ſpeedieſt way of converting intereſt into principal.

EXAMPLE III.

Borrowed on bond, 1761, June 1. the ſum of 500 l. at 5 *per cent.* and made partial payments as follows: Required the ſtate of the affair the 14th of Auguſt 1765, when the account comes to be cleared?

1761,

		L.	s.	d.
1761, June 1.	Principal borrowed -	500		
	Inter. for 1 year and 129 days	33	16	$8\frac{1}{2}$
	Amount	533	16	$8\frac{1}{2}$
1762, Oct. 8.	Paid - - -	120		
	New principal	413	16	$8\frac{1}{2}$
	Inter. for 1 year and 85 days	25	10	$2\frac{1}{4}$
	Amount	439	6	$10\frac{3}{4}$
1764, Jan. 1.	Paid - - -	239	6	$10\frac{3}{4}$
	New principal	200		
	Inter. for 1 year and 225 days	16	3	$3\frac{1}{4}$
	Amount	216	3	$3\frac{1}{4}$
1765, Aug. 14.	Paid in full - -	216	3	$3\frac{1}{4}$

PROB. II.

Amount, rate, and time in years given, to find the principal, or present worth.

RULE.

Add 1 to the product of the time and rate of one pound, by which divide the amount.

EXAMPLE I.

What ready money will pay a bill of 682 l. 18 s. $9\frac{1}{2}$ d. due $3\frac{3}{4}$ years hence, discounting interest at $4\frac{1}{2}$ *per cent.*?

$$
\begin{array}{r l}
3.75 & \text{time.} \\
.045 & \text{rate of 1 l.} \\
\hline
1875 & \\
1500 & \\
\hline
\end{array}
$$

$$
\begin{array}{l}
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \text{L.} \quad \text{s.} \quad \text{d.} \\
1.16875)682.93958\tfrac{1}{3}(584.\tfrac{1}{3} = 584 \quad 6 \quad 8 \; \textit{Anf.} \\
\qquad\quad 584375\ldots \\
\qquad\quad \overline{985645} \\
\qquad\quad 935000 \\
\qquad\quad \overline{506458} \\
\qquad\quad 467500 \\
\qquad\quad \overline{389583} \\
\qquad\quad 350625 \\
\qquad\quad \overline{{*}38958}
\end{array}
$$

Or fay, As the amount of 100 l. at the rate and time given is to 100 l. principal; fo is the given amount to the principal, or prefent worth, fought.

$$
\begin{array}{r l}
3.75 & \text{time.} \\
4.5 & \text{rate.} \\
\hline
1875 & \\
1500 & \\
\hline
16.875 & \text{intereft of 100 l. for the given time.} \\
100 & \\
\hline
116.875 & \text{amount of 100 l. for the given time.}
\end{array}
$$

Then fay, L. 116.875 : L. 100 : : L. 682.93958$\frac{1}{3}$: L. 584.$\frac{1}{3}$

If the time be given in days, divide the product of the rate and time by 365, and then proceed as before.

The difference between the prefent worth and the amount, or fum of the bill, is called *rebate*, or *difcount*; and is always lefs than the intereft of the amount for the given time, being precifely equal to the intereft of the prefent worth for the faid time; fo that if the prefent

 worth

worth be put to intereſt, it will, by the end of the given time, with the intereſt thence ariſing, be exactly equal to the given amount.

Thus, if I have a bill of 105 l. due 1 year hence, the diſcount at 5 *per cent.* will be 5 l. and the preſent worth 100 l.; for if I put the 100 l. to intereſt, it will, with the intereſt, by the end of the year, be equal to 105 l. the given amount. But the intereſt of 105 l. for a year is 5 l. 5 s.; and if this be deduced from the ſum of the bill, I ſhall then receive, as preſent worth, the ſum only of 99 l. 15 s. which is 5 s. too little.

The diſcount is found by ſubtracting the preſent worth from the amount; but it may alſo be found as follows, viz. As the amount of 100 l. or of 1 l. for the time given, to the intereſt; ſo the given amount or ſum of the bill to the diſcount ſought. The diſcount of the former example found in this manner follows.

```
116.875 : 16.875 :: 682.93958̸3
 23.375 :  3.375          .027
  4.675 :   .675   ------------
   .935 :   .135    4780577083̸
   .187 :   .027   1365879166̸6              L.  s. d.
                   ------------
           .187) 18.439368750(98.60625 = 98 12 1½
                 16 83......
                 ----
                  1609
                  1496
                  ----
                   1133
                   1122
                   ----
                     1168
                     1122
                     ----
                       467
                       374
                       ---
                        935
                        935
                        ---
```

The

The present worth is found by subtracting the discount from the amount, or sum of the bill.

L.	s.	d.	
682	18	$9\frac{1}{2}$	amount.
98	12	$1\frac{1}{2}$	discount.
584	6	8	present worth.

EXAMPLE II.

What is the present worth, and what the rebate, of a bill of 85 l. 10 s. discounting for 66 days, at 5 *per cent.*?

```
73)66.0(.9041  interest of 100 l. for 66 days.
   657
   ---
    300
    292
    ---
      80
      73
      --
       7
```

```
Then 100.9041 : .9041 :: 85.5
                         85.5
                       ------
                        45205
                       45205
                      72328
                     -------                s.  d.
100.9041)77.30055(.766 = 15  3¾ rebate.
             7063287        85 10    amount.
             -------        ---------
             6667680        84 14 8¼ present worth.
             6054246
             -------
              6134340
              6054246
              -------
              (80094)
```

EXAMPLE III.

A bill of 546 l. 16 s. dated January 10. payable the 8th of Auguſt, was preſented March 15. for diſcount at $4\frac{3}{4}$ *per cent.*: What will the rebate come to?

73)146(2	intereſt at 5 *per cent.*	March 16
.1	at $\frac{1}{4}$ *p. c.* $= \frac{1}{20}$	April 30
——		May 31
1.9	inter. of 100 l. for 146	June 30
100	days, at $4\frac{3}{4}$ *p. c.*	July 31
——		Aug. 8
101.9		——
		146 days

Then 101.9 : 1.9 :: 546.8

```
                 546.8
                   1.9
                ------
                 49212
                 5468
               -------              L.  s.  d.
101.9)1038.92(10.195 =  10   3  10¾  Anſ.
       1019
       ----
         1992
         1019
         ----
          9730
          9171
          ----
           5590
           5095
           ----
          (495)
```

Here it is to be obſerved, that an alteration, either in the days or the rate, does not produce a proportional alteration in the diſcount. Thus the diſcount of any ſum for 40 days, will be more than half the diſcount for 80 days at the ſame rate; and the diſcount at 4 *per cent.* will be more than half the diſcount of the ſame ſum for the ſame time at 8 *per cent.*

The reaſon is, because the firſt term, or diviſor, con-

ſiſts

sists of two parts; the one variable, and the other invariable. The variable part is the interest of 100 l. for the given time, which increases proportionally with the rate or time; and when this interest comes to be doubled, the dividend is of course doubled also; but the divisor being by this means increased, will not give a double quotient. Thus, $100 + 2 = 102)306(3$, but $100 + 4 = 104$ dividing 612 will not quote 6. Hence it is that tables of discount cannot be accurate, unless calculated at every different rate, and for as many days as the case may require; because every day's discount varies, being still less as the days or rate increase.

PROB. III.

Principal, amount, and time in years, given, to find the rate.

RULE.

As the product of the principal and time, to the difference of the principal and amount, or total interest; so 100 to the rate *per cent.*

EXAMPLE I.

At what rate of interest will 584 l. 6 s. 8 d. amount to 682 l. 18 s. $9\frac{1}{2}$ d. in $3\frac{3}{4}$ years?

584.3 | 682.939583
$3\frac{3}{4}$ | 584.333333
4)1753.0 | 98.606250
438.25

2191.25 : 98.60625 :: 100
438.25 : 19.72125
87.65 : 3.94425
17.53 :) 78.885(4.5 *per cent.* *Ans.*
70 12
8765
8765

If

If the time given be days, divide the product of the principal and time by 365, and then proceed as before.

EXAMPLE II.

At what rate of intereſt will 275 l. gain 8 l. 5 s. in 219 days?

```
         275
         219
        ----
        2475
        275
       550
       -----
365)60225(165 : 8.25 :: 100
    365      33 :)165(5 per cent. Anſ.
    ---          165
    2372         ---
    2190
    ----
     1825
     1825
     ----
```

PROB. IV.

Principal, amount, and rate, given, to find the time in years.

RULE.

Divide the difference of the principal and amount by the product of the principal and rate of one pound: that is, as one year's intereſt of the principal to 1 year, ſo the total intereſt to the time ſought.

EXAMPLE I.

In what time will 584 l. 6 s. 8 d. amount to 682 l. 18 s. $9\frac{1}{2}$ d. at $4\frac{1}{2}$ *per cent.*?

584.3

584.3
.045
29216
233733
26.295)98.60625(3.75 years. *Ans.*
78885 . .
197212
184065
131475
131475

682.939583
584.333333
98.606250

If the time be required in days, multiply the difference of the principal and amount by 365, and then proceed as before.

EXAMPLE II.

In how many days will 275 l. principal gain 8 l. 5 s. at 5 *per cent.*?

275
.05
13.75)3011.25(219 days. *Ans.*
2750 . .
2612
1375
12375
12375

8.25
365
4125
4950
2475
3011.25

I ſhall conclude this ſection with the explication of a rule that ſtands connected with ſimple intereſt, viz.

Equation of Payments.

When two or more debts are payable at different times, the finding a mean time, at which all the debts may be paid at once, without loſs to debtor or creditor, is

is called *equating* the terms of payment; and is commonly performed by the following

RULE.

Multiply the ſeveral debts into their reſpective times, divide the ſum of the products by the total debts, and the quot is accounted the mean time.

EXAMPLE I.

A owes B 600 l. to pay at 40 days, 200 l. at 60 days, and 200 l. at 120 days: When may theſe debts be paid at once, without injury to either party?

Debts.		*Days.*		*Prod.*
600	×	40	=	24000
200	×	60	=	12000
200	×	120	=	24000
1000				)60000(60 days. *Anſ.*

EXAMPLE II.

A owes B 640 l.; whereof 40 l. to be paid preſently, 350 l. at the end of 3 months, and 250 l. at 9 months: Required the equated time for paying the whole.

Debts.		*Mon.*		*Prod.*
40	×	0	=	0000
350	×	3	=	1050
250	×	9	=	2250
640				)3300($5\frac{5}{32}$ months. *Anſ.*

EXAMPLE III.

A owes B a certain ſum; whereof $\frac{1}{3}$ to be paid in ready money, $\frac{1}{3}$ at the end of 6 months, and the other $\frac{1}{3}$ at 8 months: Required the equated time for paying the whole.

Debts.

Debts.	Mon.	Prod.
$\frac{1}{3}$	$\times 0 =$	0
$\frac{1}{3}$	$\times 6 =$	2
$\frac{1}{3}$	$\times 8 =$	$2\frac{2}{3}$
1		$)4\frac{2}{3}(4\frac{2}{3}$ months. *Anf.*

The above method is eafy, and on that account commonly practifed, but is not accurate: for a perfon, by keeping money unpaid after it becomes due, gains the intereft thereof for that time; but by paying money before it is due, he does not lofe the intereft, as the rule fuppofes, but only the difcount thereof for that time, which is always lefs than the intereft.

They who incline to be more exact, may work by the following

RULE.

Find, by Prob. 2. the prefent worth of each debt; and then, by Prob. 4. find in what time the fum of the prefent worths will amount to the fum of the debts.

EXAMPLE IV.

A owes B 50 l.; whereof 20 l. is payable 2 years hence, and 30 l. 5 years hence: What is the equated time for paying both debts at once, difcounting intereft at 5 *per cent.*?

By Prob. 2.	L.
The prefent worth of 20 l. for 2 years is,	18.18
The prefent worth of 30 l. for 5 years is,	24.00
Sum	42.18
Total debts	50.00
Difference	7.82

By Prob. 4.

Principal	42.18
Rate	.05

2.1090)7.8200(3.7079 years.

Anſ. 3.7079 years, or 3 years and 258 days.

Some ſtill complain, that the rule laſt aſſigned is not abſolutely perfect; and argue, that the rule for finding the mean or equated time ought to be ſuch as will make the intereſt of the debts paid after they are due exactly equal to the diſcount of the debts paid before they are due.

II. *Compound Intereſt.*

Compound intereſt ariſes both from the principal and intereſt; for at the end of one year the amount or ſum of principal and intereſt becomes a new principal for the year enſuing.

The laws in Britain forbid the lending of money at compound intereſt; but then the lender may exact his intereſt at the year's end, lend it out again, and ſo, in effect, have compound intereſt on his money. Annuities too are commonly reckoned at compound intereſt; and this makes the knowledge of ſuch computations in ſome meaſure neceſſary. I propoſe, however, to be very brief, both on compound intereſt and annuities; and this the rather, becauſe operations of this ſort are performed to moſt advantage by logarithms.

In compound intereſt, the principal and the ſeveral years amounts make a ſeries of geometrical proportionals, the common ratio being the amount of 1 l. for one year, as under.

		1	2	3	4 *years.*	*Ratio,*
Pr.	1 :	1.05 :	1.1025 :	1.157625 :	1.21550625	1.05
Pr.	100 :	105 :	110.25 :	115.7625 :	121.550625	

PROB.

PROB. I.

Principal, rate, and time, given, to find the amount and intereſt.

RULE.

Multiply the amount of 1 l. for a year ſo often into itſelf as are the number of years given ſave one; and the laſt product multiplied by the principal gives the amount; from which ſubtract the principal, and the remainder is the intereſt.

EXAMPLE I.

What is the amount and intereſt of 500 l. for 3 years, at 4 *per cent.*?

```
      1.04  amount of 1 l. for a year.
      1.04
    ------
    1.0816
      1.04
  --------
  1.124864
       500  principal.       L.   s.  d.
  --------
562.432000  amount  =      562   8   7½
500
-------
 62.432     intereſt =      62   8   7½
```

Or, Multiply the given principal by the amount of 1 l. for a year, ſo often as there are years in the queſtion.

```
     500  principal given.
    1.04  amount of 1 l  for a year.
   -----
     520  amount for 1 year.
    1.04
  ------
  540.80  amount for 2 years.
    1.04
--------
562.4320  amount for 3 years.
```

Or, Multiply the given principal by the rate of 1 l. and the product is the intereſt for one year; which, added to the principal, gives the amount for the firſt year; and work in like manner for each of the remaining years.

500	principal.
.04	
20.00	intereſt.
520	amount.
.04	
20.80	intereſt.
540.80	amount.
.04	
21.6320	intereſt.
562.432	amount.

By practice, thus:

25)500	principal.
20	intereſt.
25)520	amount for 1 year.
20.8	intereſt.
25)540.8	amount for 2 years.
21.632	intereſt.
562.432	amount for 3 years.

If the given time conſiſt of years and parts, or years and days, find the intereſt of the laſt amount for the given parts or days, and add the intereſt ſo found to the laſt amount.

EXAMPLE II.

What is the amount and intereſt of 354 l. 16 s. for $3\frac{1}{2}$ years, at 5 *per cent.*?

1.05

1.05	amount of 1 l.
1.05	
1.1025	
1.05	
1.157625	
354.8	
40)410.7253500	
10.26813375	
420.99348375	amount.
354.8	
66.19348375	interest.

By practice:

20)354.8	principal.
17.74	interest.
20)372.54	amount for 1 year.
18.627	interest.
20)391.167	amount for 2 years.
19.55835	interest.
40)410.72535	amount for 3 years.
10.26813375	interest.
420.99348375	total amount.
354.8	
66.19348375	total interest.

The interest for the half-year, in the above example, is computed in the way of simple interest: but it must be observed, that the interest for the half-year found in this manner will be too much; for in simple interest the several amounts are in arithmetical proportion; but in compound interest, the amounts are in geometrical proportion; and consequently the amount of any principal at compound interest, for any number of years, will be more than at simple interest: for one year they will be equal;

equal; but for any time leſs than a year, the amount, at compound intereſt, will be leſs than at ſimple intereſt.

To compute the amount for any aliquot part of a year, in this way, obſerve the following rule, viz. Extract that root of the amount of 1 l. for a year, which is denoted by the denominator of the fraction; and the product of the principal into this root, is the amount for the part of the year required.

Thus, for $\frac{1}{4}$ of a year, extract the biquadrate root; for $\frac{1}{2}$ a year extract the ſquare root; ſo, in our example, the ſquare root of 1.05 is 1.024695; and $410.72535 \times 1.024695 = 420.868212251825$; which is ſomewhat leſs than the amount found above.

Again, for $\frac{3}{4}$ of a year, extract the biquadrate root, and raiſe this root to the third power, the numerator being 3. In like manner, if the given time be days, extract the 365th root, and raiſe this root to the power denoted by the numerator, or number of days given. But ſuch extractions and involutions will prove a troubleſome taſk, without the help of logarithms.

PROB. II.

Amount, rate, and time, given, to find the principal or preſent worth.

RULE.

Divide the given amount by the amount of 1 l. for the given time and rate, and the quot is the principal or preſent worth; that is, as the amount of 1 l. to 1 l. principal, ſo the given amount to the principal ſought.

EXAMPLE.

What ready money will pay a debt of 562.432 l. due 3 years hence, diſcounting at 4 *per cent.* compound intereſt?

```
          1.04
          1.04
        ------
        1.0816
          1.04
      --------
1.124864)562.432000(500 l.   Ans.
         5624320
        --------
```

The difference of the given amount and present worth is the discount or rebate, which in the above example is 62.432 l.

If the given time be less than a year, the present worth may be found nearly in the way of simple interest; but, to be accurate, find the amount of 1 l. for the given time, in the way taught above, by which divide the given amount.

PROB. III.

Principal, amount, and time, given, to find the rate.

RULE.

Divide the amount by the principal, and that root of the quot which is denoted by the number of years will be the amount of 1 l. for a year; from which subtract 1, and there will remain the rate.

EXAMPLE.

At what rate of compound interest will 500 l. amount to 562.432 in 3 years?

500)562.432(1.124864; and the cube root of 1.124864 is 1.04, and hence .04 is the rate of 1 l.; that is, 4 *per cent*.

PROB. IV.

Principal, amount, and rate, given, to find the time.

RULE.

RULE.

Divide the amount by the principal, and again divide the quot by the amount of 1 l. for a year continually till the quot is 1, and the number of continual divisions is the number of years. Or, Raise the amount of 1 l. in a year to a power equal to the first quot, and the index of that power is the time in years.

EXAMPLE.

In what time will 500 l. amount to 562.432 l. at 4 *per cent.* compound interest?

500)562.4320	Or, 1.04
1.04)1.124864	1.04
1.04)1.0816	1.0816 square.
1.04)1.04	1.04
	1.124864 cube. *Ans.* 3 years.

For ease and dispatch in calculations of compound interest, tables, framed for years and for days at all the different rates, are absolutely necessary, and accordingly are universally used. I shall here subjoin three tables of this kind, and then show their construction and use.

TABLE

TABLE I.

The amount of 1 l. for years, compound intereſt.

Y.	2 *per cent.*	$2\frac{1}{2}$ *per cent.*	3 *per cent.*	$3\frac{1}{2}$ *per cent.*
1	1.0200000	1.0250000	1.0300000	1.0350000
2	1.0404000	1.0506250	1.0609000	1.0712250
3	1.0612080	1.0768906	1.0927270	1.1087178
4	1.0824321	1.1038128	1.1255088	1.1475230
5	1.1040808	1.1314082	1.1592740	1.1876863
6	1.1261624	1.1596934	1.1940523	1.2292553
7	1.1486856	1.1886857	1.2298738	1.2722792
8	1.1716593	1.2184029	1.2667700	1.3168089
9	1.1950925	1.2488629	1.3047731	1.3628973
10	1.2189944	1.2800845	1.3439163	1.4105987
11	1.2433743	1.3120866	1.3842338	1.4599697
12	1.2682417	1.3448888	1.4257608	1.5110686
13	1.2936066	1.3785110	1.4685337	1.5639560
14	1.3194787	1.4129738	1.5125897	1.6186945
15	1.3458683	1.4482981	1.5579674	1.6753488
16	1.3727857	1.4845056	1.6047064	1.7339860
17	1.4002414.	1.5216182	1.6528476	1.7946755
18	1.4282462	1.5596587	1.7024330	1.8574892
19	1.4568111	1.5986501	1.7535060	1.9225013
20	1.4859474	1.6386164	1.8061112	1.9897888
21	1.5156663	1.6795818	1.8602945	2.0594314
22	1.5459796	1.7215714	1.9161034	2.1315115
23	1.5768992	1.7646106	1.9735865	2.2061144
24	1.6084372	1.8087259	2.0327941	2.2833284
25	1.6406059	1.8539441	2.0937779	2.3632449

TABLE I.

The amount of 1 l. for years, compound intereſt.

Y.	2 *per cent.*	$2\frac{1}{2}$ *per cent.*	3 *per cent.*	$3\frac{1}{2}$ *per cent.*
26	1.6734181	1.9002927	2.1565912	2.4459585
27	1.7068864	1.9478000	2.2212890	2.5315671
28	1.7410242	1.9964950	2.2879276	2.6201719
29	1.7758446	2.0464073	2.3565655	2.7118779
30	1.8113615	2.0975675	2.4272624	2.8067937
31	1.8475888	2.1500067	2.5000803	2.9050314
32	1.8845405	2.2037569	2.5750827	3.0067075
33	1.9222314	2.2588508	2.6523352	3.1119423
34	1.9606760	2.3153221	2.7319053	3.2208603
35	1.9998895	2.3732051	2.8138624	3.3335904
36	2.0398873	2.4325353	2.8982783	3.4502661
37	2.0806850	2.4933487	2.9852266	3.5710254
38	2.1222987	2.5556824	3.0747834	3.6960113
39	2.1647447	2.6195744	3.1670269	3.8253717
40	2.2080396	2.6850638	3.2620377	3.9592597
41	2.2522004	2.7521904	3.3598989	4.0978338
42	2.2979444	2.8209952	3.4606958	4.2412579
43	2.3431893	2.8915500	3.5645167	4.3897020
44	2.3900531	2.9638080	3.6714522	4.5433416
45	2.4378542	3.0379032	3.7815958	4.7023585
46	2.4866112	3.1138508	3.8950437	4.8669411
47	2.5363435	3.1916971	4.0118950	5.0372840
48	2.5870703	3.2714895	4.1322518	5.2135889
49	2.6388117	3.3532768	4.2562194	5.3960645
50	2.6015880	3.4371087	4.3839060	5.5849268

TABLE

TABLE I.

The amount of 1 l. for years, compound intereft.

Y.	4 *per cent.*	$4\frac{1}{2}$ *per cent.*	5 *per cent.*	6 *per cent.*
1	1.0400000	1.0450000	1.0500000	1.0600000
2	1.0816000	1.0920250	1.1025000	1.1236000
3	1.1248640	1.1411661	1.1576250	1.1910160
4	1.1698586	1.1925186	1.2155063	1.2624769
5	1.2166529	1.2461819	1.2762816	1.3382256
6	1.2653190	1.3022601	1.3400956	1.4185191
7	1.3159318	1.3608618	1.4071034	1.5036303
8	1.3685691	1.4221006	1.4774554	1.5938481
9	1.4233118	1.4860951	1.5513282	1.6894790
10	1.4802443	1.5529694	1.6288946	1.7908477
11	1.5394541	1.6228530	1.7103393	1.8982980
12	1.6010322	1.69588814	1.7958563	2.0121965
13	1.6650735	1.7721961	1.8856491	2.1329283
14	1.7316764	1.8519449	1.9799316	2.2609039
15	1.8009435	1.9352824	2.0789282	2.3965582
16	1.8729812	2.0223701	2.1828746	2.5402517
17	1.9479005	2.1133768	2.2920183	2.6927728
18	2.0258165	2.2084787	2.4066192	2.8543392
19	2.1068492	2.3078603	2.5269502	3.0255995
20	2.1911231	2.4117140	2.6532977	3.2071355
21	2.2787681	2.5202411	2.7859626	3.3995636
22	2.3699188	2.6336520	2.9252607	3.6035374
23	2.4647155	2.7521663	3.0715238	3.8197497
24	2.5633042	2.8760138	3.2251000	4.0489346
25	2.6658363	3.0054344	3.3863549	4.2918707

TABLE I.

The amount of 1 l. for years, compound intereſt.

Y.	4 per cent	4½ per cent	5 per cent.	6 per cent.
26	2.7724697	3.1406790	3.5556727	4.5493829
27	2.8833685	3.2820095	3.7334563	4.8223459
28	2.9987033	3.4296999	3.9201291	5.1116866
29	3.1186514	3.5840364	4.1161356	5.4183878
30	3.2433975	3.7453181	4.3219424	5.7434911
31	3.3731334	3.9138574	4.5380395	6.0881006
32	3.5080587	4.0899810	4.7649415	6.4533866
33	3.6483811	4.2740301	5.0031885	6.8405898
34	3.7943163	4.4663615	5.2533480	7.2510252
35	3.9460889	4.6673478	5.5160154	7.6860867
36	4.1039325	4.8773784	5.7918161	8.1472519
37	4.2680898	5.0968604	6.0814069	8.6360870
38	4.4388134	5.3262192	6.3854773	9.1542523
39	4.6163659	5.5658990	6.7047511	9.7035074
40	4.8010206	5.8163645	7.0399887	10.2857178
41	4.9930614	6.0781009	7.3919881	10.9028609
42	5.1927839	6.3516154	7.7615875	11.5570326
43	5.4004952	6.6374381	8.1496669	12.2504545
44	5.6165150	6.9361229	8.5571503	12.9854818
45	5.8411756	7.2482484	8.9850078	13.7646107
46	6.0748227	7.5744196	9.4342582	14.5904873
47	6.3178156	7.9152684	9.9059711	15.4659166
48	6.5705282	8.2714555	10.4012696	16.3938716
49	6.8333493	8.6436710	10.9213331	17.3775039
50	7.1066833	9.0326362	11.4674000	18.4201541

TABLE

TABLE II.

The amount of 1 l. for days, compound intereſt.

D.	2 *per cent.*	$2\frac{1}{2}$ *per cent.*	3 *per cent.*	$3\frac{1}{2}$ *per cent.*
1	1.0000542	1.0000676	1.0000809	1.0000942
2	1.0001085	1.0001353	1.0001619	1.0001885
3	1.0001627	1.0002029	1.0002429	1.0002827
4	1.0002170	1.0002706	1.0003240	1.0003770
5	1.0002713	1.0003383	1.0004050	1.0004713
6	1.0003255	1.0004059	1.0004860	1.0005656
7	1.0003798	1.0004736	1.0005670	1.0006600
8	1.0004341	1.0005412	1.0006480	1.0007542
9	1.0004884	1.0006090	1.0007291	1.0008486
10	1.0005426	1.0006767	1.0008101	1.0009429
20	1.0010856	1.0013539	1.0016209	1.0018867
30	1.0016289	1.0020315	1.0024324	1.0028315
40	1.0021725	1.0027097	1.0032445	1.0037771
50	1.0027163	1.0033882	1.0040573	1.0047236
60	1.0032605	1.0040673	1.0048708	1.0056710
70	1.0938049	1.0047468	1.0056849	1.0066193
80	1.0043497	1.0054267	1.0064996	1.0075685
90	1.0048947	1.0061071	1.0073151	1.0085186
100	1.0054401	1.0067880	1.0081311	1.0094696
110	1.0059857	1.0074693	1.0089479	1.0104214
120	1.0065316	1.0081511	1.0097653	1.0113742
130	1.0070779	1.0088334	1.0105834	1.0123279
140	1.0076244	1.0095161	1.0114021	1.0132825
150	1.0081712	1.0101993	1.0122215	1.0142379
160	1.0087183	1.0108829	1.0130415	1.0151943

TABLE

TABLE II.

The amount of 1 l. for days, compound intereſt.

D.	2 per cent.	$2\frac{1}{2}$ per cent.	3 per cent.	$3\frac{1}{2}$ per cent.
170	1.0092658	1.0115670	1.0138623	1.0161516
180	1.0098135	1.0122516	1.0146837	1.0171098
190	1.0103615	1.0129366	1.0155057	1.0180689
200	1.0109098	1.0136221	1.0163284	1.0190288
210	1.0114584	1.0143081	1.0171518	1.0199897
220	1.0120073	1.0149945	1.0179759	1.0209515
230	1.0125565	1.0156814	1.0188006	1.0219142
240	1.0131000	1.0163687	1.0196260	1.0228778
250	1.0136558	1.0170565	1.0204520	1.0238424
260	1.0142059	1.0177448	1.0212788	1.0248078
270	1.0147563	1.0184336	1.0221062	1.0257741
280	1.0153070	1.0191228	1.0229342	1.0267414
290	1.0158580	1.0198125	1.0237630	1.0277096
300	1.0164093	1.0205026	1.0245924	1.0286386
310	1.0169609	1.0211932	1.0254225	1.0296486
320	1.0175127	1.0218843	1.0262532	1.0306195
330	1.0180649	1.0225758	1.0270847	1.0315914
340	1.0186174	1.0232679	1.0279168	1.0325641
350	1.0191702	1.0239603	1.0287495	1.0335378
360	1.0197233	1.0246533	1.0295830	1.0345123
361	1.0197786	1.0247226	1.0296664	1.0346098
362	1.0198340	1.0247919	1.0297497	1.0347073
363	1.0198893	1.0248613	1.0298331	1.0348049
364	1.0199446	1.0249306	1.0299165	1.0349024
365	1.0200000	1.0250000	1.0300000	1.0350000

TABLE

TABLE II.

The amount of 1 l. for days, compound interest.

D.	4 *per cent.*	4½ *per cent.*	5 *per cent.*	6 *per cent.*
1	1.0001074	1.0001206	1.0001336	1.0001596
2	1.0002149	1.0002412	1.0002673	1.0003193
3	1.0003224	1.0003618	1.0004011	1.0004790
4	1.0004299	1.0004824	1.0005348	1.0006387
5	1.0005374	1.0006031	1.0006685	1.0007985
6	1.0006449	1.0007238	1.0008023	1.0009583
7	1.0007524	1.0008445	1.0009361	1.0011181
8	1.0008600	1.0009652	1.0010699	1.0012779
9	1.0009675	1.0010859	1.0012037	1.0014378
10	1.0010751	1.0012066	1.0013376	1.0015976
20	1.0021513	1.0024148	1.0026770	1.0031979
30	1.0032288	1.0036243	1.0040182	1.0048007
40	1.0043074	1.0048354	1.0053611	1.0064060
50	1.0053871	1.0060479	1.0067059	1.0080139
60	1.0064680	1.0072618	1.0080525	1.0096244
70	1.0075501	1.0084773	1.0094009	1.0112375
80	1.0086333	1.0096942	1.0107511	1.0128531
90	1.0097177	1.0109125	1.0121031	1.0144713
100	1.0108033	1.0121324	1.0134569	1.0160921
110	1.0118900	1.0133537	1.0148125	1.0177155
120	1.0129779	1.0145765	1.0161699	1.0193415
130	1.0140670	1.0158007	1.0175291	1.0209701
140	1.0151572	1.0170265	1.0188902	1.0226013
150	1.0162487	1.0182537	1.0202531	1.0242351
160	1.0173412	1.0194824	1.0216178	1.0258715

TABLE II.

The amount of 1 l. for days, compound intereſt.

D.	4 *per cent.*	$4\frac{1}{2}$ *per cent.*	5 *per cent.*	6 *per cent.*
170	1.0184350	1.0207120	1.0229843	1.0275105
180	1.0195299	1.0219442	1.0243527	1.0291522
190	1.0206261	1.0231774	1.0257228	1.0307964
200	1.0217233	1.0244120	1.0270949	1.0324433
210	1.0228218	1.0256481	1.0284687	1.0340928
220	1.0239215	1.0268858	1.0298444	1.0357450
230	1.0250233	1.0281249	1.0312219	1.0373998
240	1.0261243	1.0293655	1.0326013	1.0390572
250	1.0272275	1.0306076	1.0339825	1.0407173
260	1.0283319	1.0318512	1.0353656	1.0423800
270	1.0294375	1.0330963	1.0367505	1.0440454
280	1.0305443	1.0343429	1.0381373	.0457135
290	1.0316522	1.0355910	1.0395259	1.0473842
300	1.0327614	1.0368406	1.0409164	1.0490576
310	1.0338717	1.0383917	1.0423087	1.0507336
320	1.0349832	1.0393444	1.0437029	1.0524124
330	1.0360960	1.0405985	1.0450990	1.0540938
340	1.0372099	1.0418542	1.0464969	1.0557779
350	1.0383250	1.0431114	1.0478967	.0574647
360	1.0394413	1.0443700	1.0492984	1.0591542
361	1.0395530	1.0444960	1.0494387	.0593233
362	1.0396648	1.0446220	1.0495790	1.0594924
363	1.0397765	1.0447479	1.0497193	1.0596616
364	1.0398882	1.0448739	1.0498596	1.0598308
365	1.0400000	1.0450000	1.0500000	1.0600000

TABLE

TABLE III.

The present worth of 1 l. for years, compound interest.

Y.	2 per c.	2½ per c.	3 per c.	3½ per c.
1	.9803921	.9756097	.9708738	.9661836
2	.9611687	.9518144	.9425959	.9335107
3	.9423223	.9285994	.9151417	.9019427
4	.9238454	.9059506	.8884870	.8714422
5	.9057308	.8838542	.8626088	.8419732
6	.8879713	.8622968	.8374843	.8135006
7	.8705601	.8412654	.8130915	.7859910
8	.8534903	.8207465	.7894092	.7594116
9	.8367552	.8007283	.7664167	.7337310
10	.8203483	.7811984	.7440939	.7089188
11	.8042630	.7621447	.7224213	.6849457
12	.7884931	.7435558	.7013799	.6617833
13	.7730325	.7254203	.6809513	.6394041
14	.7578750	.7077272	.6611178	.6177818
15	.7430147	.6904655	.6418619	.5968906
16	.7284458	.6736249	.6231669	.5767059
17	.7141625	.6571950	.6050164	.5572038
18	.7001593	.6411659	.5873946	.5383611
19	.6864307	.6255277	.5702860	.5201557
20	.6729713	.6102709	.5536758	.5025659
21	.6597758	.5953802	.5375493	.4855709
22	.6468390	.5808646	.5218925	.4691506
23	.6341559	.5666972	.5066917	.4532856
24	.6217214	.5528753	.4919337	.4379571
25	.6095308	.5393905	.4776056	.4231470

TABLE III.

The present worth of 1 l. for years, compound interest.

Y.	2 *per c.*	$2\frac{1}{2}$ *per c.*	3 *per c.*	$3\frac{1}{2}$ *per c.*
26	.5975793	.5262347	.4636947	.4088378
27	.5858620	.5133997	.4501891	.3950123
28	.5743746	.5008778	.4370768	.3816543
29	.5631123	.4886613	.4243464	.3687482
30	.5520709	.4767427	.4119868	.3562784
31	.5412460	.4651148	.3999871	.3442304
32	.5306333	.4537706	.3883370	.3325897
33	.5202287	.4427030	.3770263	.3213427
34	.5100282	.4319053	.3660449	.3104761
35	.5000276	.4213711	.3553834	.2999769
36	.4902232	.4110937	.3450324	.2898327
37	.4806109	.4010671	.3349829	.2800316
38	.4711872	.3912849	.3252262	.2705619
39	.4619482	.3817414	.3157536	.2614125
40	.4528904	.3724306	.3065568	.2525725
41	.4440102	.3633470	.2976280	.2440314
42	.4353041	.3544848	.2889592	.2357791
43	.4267688	.3458389	.2805429	.2278059
44	.4184008	.3374038	.2723718	.2201023
45	.4101968	.3291744	.2644386	.2126594
46	.4021537	.3211458	.2567365	.2054679
47	.3942084	.3133129	.2492588	.1985197
48	.3865376	.3056712	.2419988	.1918065
49	.3789584	.2982158	.2349503	.1853202
50	.[illegible]15279	.2[illegible]422	.2281071	.1790534

TABLE

TABLE III.

The present worth of 1 l. for years, compound interest.

Y.	4 per c.	4½ per c.	5 per c.	6 per c.
1	.9615385	.9569378	.9523810	.9433962
2	.9245562	.9157299	.9070295	.8899964
3	.8889964	.8762966	.8638376	.8396193
4	.8548042	.8385613	.8227025	.7920937
5	.8219271	.8024511	.7835262	.7472582
6	.7903145	.7678957	.7462154	.7049605
7	.7599178	.7348285	.7106813	.6650571
8	.7306902	.7031851	.6768394	.6274124
9	.7025867	.6729044	.6446089	.5918985
10	.6755642	.6439277	.6139133	.5583948
11	.6495809	.6161987	.5846793	.5267875
12	.6245971	.5896639	.5568374	.4969694
13	.6005741	.5642716	.5303214	.4688390
14	.5774751	.5399729	.5050679	.4423010
15	.5552645	.5167204	.4810171	.4172651
16	.5339082	.4944693	.4581115	.3936463
17	.5133733	.4731764	.4362967	.3713644
18	.4936281	.4528004	.4155207	.3503438
19	.4746424	.4333018	.3957340	.3305130
20	.4563870	.4146429	.3768895	.3118047
21	.4388336	.3967874	.3589424	.2941554
22	.4219554	.3797009	.3418499	.2775051
23	.4057263	.3633501	.3255713	.2617973
24	.3901215	.3477035	.3100679	.2469786
25	.3751168	.3327306	.2953028	.2329986

TABLE III.

The preſent worth of 1 l. for years, compound intereſt.

Y.	4 per c.	4½ per c.	5 per c.	6 per c.
26	.3606892	.3184025	.2812407	.2198100
27	.3468166	.3046914	.2678483	.2073680
28	.3334775	.2915707	.2550936	.1956301
29	.3206514	.2790150	.2429463	.1845567
30	.3083187	.2670000	.2313775	.1741101
31	.2964603	.2555024	.2203595	.1642548
32	.2850579	.2444999	.2098662	.1549574
33	.2740942	.2339712	.1998726	.1461862
34	.2635521	.2238959	.1903548	.1379115
35	.2534155	.2142544	.1812903	.1301052
36	.2436687	.2050282	.1726574	.1227408
37	.2342969	.1961992	.1644356	.1157932
38	.2252854	.1877504	.1566054	.1092389
39	.2166206	.1796655	.1491479	.1030555
40	.2082890	.1719287	.1420457	.0972222
41	.2002779	.1645251	.1352816	.0917191
42	.1925749	.1574403	.1288396	.0865274
43	.1851682	.1506605	.1227044	.0816296
44	.1180464	.1441728	.1168613	.0770091
45	.1711984	.1379644	.1112965	.0726501
46	.1646139	.1320233	.1059967	.0685378
47	.1582826	.1263381	.1009492	.0646583
48	.1521948	.1208977	.0961421	.0609984
49	.1463411	.1156916	.0915639	.0575457
50	.1407126	.1107097	.0872037	.0542884

Table

Table I. ſhews the amount of 1 l. principal for years; and is conſtructed by the rule aſſigned in Prob. 1 *viz.* Multiply the amount of 1 l. for a year by itſelf continually. Thus, at 4 *per cent.* the amount of 1 l. is 1.04 for 1 year; and 1.04 × 1.04 = 1.0816 for 2 years; and 1.0816 × 1.04 = 1.124864 for 3 years, &c.

Table II. ſhews the amount of 1 l. for days; to conſtruct which, find, by the directions given in the end of Prob. 1. the amount of 1 l. for one day; and this amount multiplied into itſelf continually, will give the amount for any number of days.

Example at 5 per cent.

	1.00013368	the amount for 1 day,
	1.00013368	
Prod.	1.00026738	the amount for 2 days.
	1.00013368	
Prod.	1.00040110	the amount for 3 days.

Table III. ſhews the preſent worth of 1 l. principal for years; and is conſtructed by the rule aſſigned in Prob. 2. viz. Divide 1 l. by the amount of 1 l. for the given time and rate, as contained in Table I.

Example at 4 per cent.

1.04)1(.9615385 for 1 year.
1.0816)1(.9245562 for 2 years.
1.124864)1(.8889964 for 3 years, &c.

The table may, in this manner, be continued to any number of years whatever.

The uſe of Table I.

1. Principal, rate, and time, given, to find the amount.
Multiply the given principal by the tabular number correſponding to the rate and time.

Ex.

Ex. What will 500 l. principal amount to in 15 years, at 5 *per cent.* compound intereſt?

2.0789282			
500			
	L.	*s.*	*d.*
1039.4641000 =	1039	9	$3\frac{1}{4}$

If the given principal conſiſt of pounds, ſhillings, pence, reduce the ſhillings and pence to a decimal, and then proceed as above.

2. Principal, amount, and time, given, to find the rate.

Divide the amount by the principal, and the quot will be the amount of 1 l.; which find in the table even with the given time, and you have the rate on the head.

Ex. At what rate of intereſt will 500 l. amount to 1039.4641 l. in 15 years?

500)1039.4641

2.0789282 found under 5 *per cent.*

3. Principal, amount, and rate, given, to find the time.

Divide the amount by the principal, and the quot will be the amount of 1 l.; which find under the given rate, and on the ſide you have the time.

Ex. In what time will 500 l. amount to 1039.4641 l. at 5 *per cent.* compound intereſt?

500)1039.4641

2.0789282 found oppoſite to 15 years.

4. Amount, rate, and time, given, to find the principal.

Divide the given amount by the tabular amount of 1 l. for the rate and time given, and the quot will be the principal.

Ex.

Ex. What principal will amount to 1039.4641 l. in 15 years, at 5 *per cent.*?

2.0789282)1039.4641(500 principal.

The use of Table II.

This table shews the amount of 1 l. for days, and is used in the same manner as the former.

But if the given number of days be not in this table; or if the given number of years be not in Table I. work as follows.

Divide the given number of days or years into two such numbers as are found in the tables, multiply the tabular amounts into one another, and their product is the amount of 1 l. for the time given.

Ex. What will 500 l. amount to in 75 years and 184 days, at 5 *per cent.*?

Amount for 50 years, at 5 *per cent.* is,	11.4674000
Amount for 25 years, at 5 *per cent.* is,	3.3863549
Their product is the amount for 75 years,	38.8326862
Amount for 180 days, is, —	1.0243527
Amount for 4 days, is, —	1.0005348
Their product is the amount for 184 days,	1.0249005
Amount for 75 years, is, —	38.8326862
Amount for 184 days, is, —	1.0249005
Their product is the amount of 1 l. for 75 years and 184 days,	39.7996395
Multiply by the principal,	500
Amount sought, — —	19899.8197500

And so, 500 l. in 75 years and 184 days, at 5 *per cent.* will amount to 19899 l. 16 s. 4½ d. And in this manner may these two tables be extended to any number of years and days.

The

The use of Table III.

Amount, rate, and time, given, to find the principal or present worth.

Multiply the tabular number answering to the rate and time, by the given amount.

Ex. What ready money will pay a debt of 562.432 l. due 3 years hence, discounting at 4 *per cent.* compound interest?

.8889964 × 562.432 = 500 l. the present worth sought.

I shall here subjoin two or three practical questions.

Quest. 1. What will L. 136 : 15 : 6 amount to in 20 years, at 6 *per cent.* compound interest?

By Table I.

```
3.2071355  amount of 1 l.
  577.631  multiplier inverted.
---------
32071355
 9621406
 1924281
  224499
   22449
    1603
---------
438.65593  amount sought.
```

Quest. 2. What will 5 l. amount to in 400 years, at 5 *per cent.* compound interest?

By

By Table I.

The amount of 1 l. is,	11.4674	for 50 years.
	11.4674	
	131.5013	for 100 years.
	131.5013	
	17292.5919	for 200 years.
	17292.5919	
	297650327.2679	for 400 years.
	5	
Amount of 5 l. is,	1488251636.3395	for 400 years.

The above example ſhows the rapid ſurpriſing increaſe of numbers in geometrical proportion; and for the learner's amuſement, it may not be improper to obſerve, that if a ſingle farthing had been lent on compound intereſt, at 5 *per cent.* in the firſt year of the Chriſtian æra, or at the birth of Chriſt, and had continued to this preſent year 1766, the amount would have been ſuch an immenſe ſum as to ſurpaſs in value ſome millions of globes of ſolid gold, each as large as this earth.

Queſt. 3. A owes B ſeveral ſums, viz. 180 l. due 3 years hence; 200 l. due 5 years hence; and 300 l. due 7 years hence: What is the preſent worth of theſe debts, diſcounting at 4 *per cent.* compound intereſt?

By Table III.

$$.8889964 \times 180 = 160.0193520$$
$$.8219271 \times 200 = 164.3854200$$
$$.7599178 \times 300 = 227.9753400$$

Preſent worth, 552.380112

CHAP. XIII.

ANNUITIES.

AN annuity is a ſum of money, payable yearly, half-yearly, or quarterly, to continue a certain number of years, for ever, or for life.

An annuity is ſaid to be in arrear, when it continues unpaid after it falls due. And an annuity is ſaid to be in reverſion, when the purchaſer, upon paying the price, does not immediately enter upon poſſeſſion; the annuity not commencing till ſome time after.

Intereſt on annuities may be computed either in the way of ſimple or compound intereſt. But compound intereſt being found moſt equitable, both for buyer and ſeller, the computation by ſimple intereſt is univerſally diſuſed.

I. *Annuities for a certain time.*

PROB. I.

Annuity, rate, and time, given, to find the amount, or ſum of yearly payments, and intereſt.

RULE.

Make 1 the firſt term of a geometrical ſeries, and the amount of 1 l. for a year the common ratio; continue this ſeries to as many terms as there are years in the queſtion; and the ſum of this ſeries is the amount of 1 l. annuity for the given years; which, multiplied by the given annuity, will produce the amount ſought.

EXAMPLE.

An annuity of 40 l. payable yearly, is forborn and unpaid till the end of 5 years: What will then be due, reckoning compound intereſt at 5 *per cent.* on all the payments then in arrear?

1 : 2 : 3 : 4 : 5
1 : 1.05 : 1.1025 : 1.157625 : 1.21550625; whose sum is 5.52563125 l.; and 5.52563125 × 40 = 221.02525 = 221 l. 0 s. 6 d. the amount sought.

The amount may also be found thus: Multiply the given annuity by the amount of 1 l. for a year; to the product add the given annuity, and the sum is the amount in 2 years; which multiply by the amount of 1 l. for a year; to the product add the given annuity, and the sum is the amount in 3 years, &c. The former question wrought in this manner follows.

```
  40 am. in 1 year.          126.1 am. in 3 years.
 1.05                          1.05
 -----                        ------
 42.00                        132.405
 40                            40
 -----                        --------
 82 am. in 2 years.           172.405 am. in 4 years.
1.05                             1.05
 -----                       ---------
 86.10                        181.02525
 40                            40
 -----                       ----------
126.1 am. in 3 years.        221.02525 am. in 5 years.
```

If the given time be years and quarters, find the amount for the whole years, as above; then find the amount of 1 l. for the given quarters; by which multiply the amount for the whole years; and to the product add such a part of the annuity as the given quarters are of a year.

If the given annuity be payable half-yearly, or quarterly, find the amount of 1 l. for half a year or a quarter; by which find the amount for the several half-years or quarters, in the same manner as the amount for the several years is found above.

PROB. II.

Annuity, rate, and time, given, to find the present worth, or sum of money that will purchase the annuity.

 RULE.

RULE.

Find the amount of the given annuity by the former problem; and then, by Prob. 2. in compound intereſt, find the preſent worth of this amount, as a ſum due at the end of the given time.

EXAMPLE.

What is the preſent worth of an annuity of 40 l. to continue 5 years, diſcounting at 5 *per cent.* compound intereſt?

By the former problem, the amount of the given annuity for 5 years, at 5 *per cent.* is 221.02525; and by Prob. 1. in compound intereſt, the amount of 1 l. for 5 years, at 5 *per cent.* is 1.2762815625

And, 1.2762815625)221.02525000(173.179 = 173 l. 3 s. 7 d. the preſent worth ſought.

The preſent worth may alſo be found thus: By Prob. 2. of compound intereſt, find the preſent worth of each year by itſelf, and the ſum of theſe is the preſent worth ſought. The former example done in this way follows.

1.2762815625)40.0000000000	(31.3410
1.21550625)40.0000000	(32.9080
1.157625)40.00000	(34.5535
1.1025)40.000	(36.2811
1.05)40.0	(38.0952
Preſent worth,	173.1788

A third way of finding the preſent worth, take as follows, viz. Find a principal ſum, whereof the given annuity is one year's intereſt; then find the preſent worth of this principal, as a ſum due at the end of the given time; ſubtract this preſent worth from its principal; and the remainder is the preſent worth ſought. The former queſtion done in this manner follows.

As

As .05 : 1 :: 40 : 800 l. principal.

$1.05 \times 1.05 \times 1.05 \times 1.05 \times 1.05 = 1.2762815625$

And 1.2762815625)800.00000000(626.8208

$$\begin{array}{r} 800 \\ 626.8208 \\ \hline 173.1792 \end{array}$$ present worth sought.

If the annuity to be purchased be in reversion, find first the present worth of the annuity, as commencing immediately, by any of the three methods taught above; and then, by Prob. 2. of compound interest, find the present worth of that present worth, rebating for the time in reversion; and this last present worth is the answer.

Ex. What is the present worth of a yearly pension or rent of 75 l. to continue 4 years, but not to commence till 3 years hence, discounting at 5 *per cent.*?

.05 : 1 :: 75 : 1500

$1.05 \times 1.05 \times 1.05 \times 1.05 = 1.21550625$

1.21550625)1500.00000(1234.05371

$$\begin{array}{r} 1500 \\ 1234.05371 \\ \hline 265.94629 \end{array}$$ present worth of the annuity, if it was to commence immediately.

$1.05 \times 1.05 \times 1.05 = 1.157625$ L. s. d.

1.157625)265.94629(229.7344 = 229 14 $8\frac{1}{4}$

PROB. III.

Present worth, rate, and time, given, to find the annuity.

RULE.

By the preceding problem, find the present worth of 1 l. annuity for the rate and time given; and then say, As the present worth thus found to 1 l. annuity, so the present worth given to its annuity; that is, divide the given present worth by that of 1 l. annuity.

EX.

EXAMPLE.

What annuity, to continue 5 years, will 173 l. 3 s. 7 d. purchase, allowing compound interest at 5 *per cent.*?

.05 : 1 :: 1 : 20 l.

1.05 × 1.05 × 1.05 × 1.05 × 1.05 = 1.2762815625
1.2762815625)20.000000000(15.6705

20
15.6705

4.3295 present worth of 1 l. annuity.
4.329)173.179(40 l. annuity. *Ans.*

The operations in annuities, as well as in compound interest, turn out heavy and troublesome; and on that account tables are universally used. I shall here annex three tables of that sort; and to the tables shall subjoin a brief illustration of their uses. And in regard there will sometimes be occasion for the promiscuous use of these tables on annuities, and the three former tables on compound interest, the following tables shall be numbered Table IV. V. VI.; by which means any table to be used will be easily referred to.

Table IV. shows the amount of 1 l. annuity, and may be constructed from Prob. I.; or rather thus: To 1 l. the first year of this table, add the first year of Table I. and the sum is the second year of this table; to which add the second year of Table I. and the sum will be the third year of this table, &c. The reason is obvious.

Table V. shews the present worth of 1 l. annuity; and may be constructed from Prob. II. or more easily from Table III.; thus: The first year in Table III. and V. is the same; the sum of the first and second years in Table III. make the second year in Table V.; and the sum of the second and third years in Table III. make the third year in Table V. &c.

Table VI. shews the annuity which 1 l. will purchase; and may be constructed from Prob. III.; but more readily thus: Divide 1 by the first year of Table V. and the quot is the first year of Table VI. Again, divide 1 by the second year of Table V. and the quot is the second year of Table VI. &c.

TABLE

TABLE IV.

The amount of 1 l. annuity, compound intereſt.

Y.	2 *per cent.*	$2\frac{1}{2}$ *per cent.*	3 *per cent.*	$3\frac{1}{2}$ *per cent.*
1	1.0000000	1.0000000	1.0000000	1.0000000
2	2.0200000	2.0250000	2.0300000	2.0350000
3	3.0604000	3.0756230	3.0909000	3.1062250
4	4.1216080	4.1525156	4.1836270	4.2149429
5	5.2040402	5.2563285	5.3091358	5.3624659
6	6.3081210	6.3877367	6.4684099	6.5501522
7	7.4342834	7.5474302	7.6624622	7.7794075
8	8.5829691	8.7361159	8.8923360	9.0516866
9	9.7546284	9.9545188	10.1591061	10.3684958
10	10.9497210	11.2033818	11.4638793	11.7313931
11	12.1687154	12.4834663	12.8077957	13.1419919
12	13.4120897	13.7955530	14.1920296	14.6019616
13	14.6803315	15.1404418	15.6177904	16.1130303
14	15.9739381	16.5189528	17.0863242	17.6769864
15	17.2934169	17.9319267	18.5989139	19.2956809
16	18.6392853	19.3802248	20.1568813	20.9710297
17	20.0120719	20.8647304	21.7615877	22.7050158
18	21.4123124	22.3863487	23.4144354	24.4996913
19	22.8405586	23.9460074	25.1168684	26.3571805
20	24.2973698	25.5446576	26.8703745	28.2796818
21	25.7833172	27.1832740	28.6764857	30.2694707
22	27.2989835	28.8628559	30.5367803	32.3289022
23	28.8449632	30.5844273	32.4528837	34.4604137
24	30.4218625	32.3490379	34.4264702	36.6665282
25	32.0302997	34.1577639	36.4592643	38.9498567

TABLE IV.

The amount of 1 l. annuity, compound intereſt.

Y.	2 *per cent.*	2½ *per cent.*	3 *per cent.*	3½ *per cent.*
26	33.6709057	36.0117080	38.5530422	41.3131017
27	35.3443238	37.9120007	40.7096335	43.7590602
28	37.0512103	39.8598008	42.9309225	46.2906273
29	38.7922345	41.8562958	45.2188502	48.9107993
30	40.5680792	43.9027032	47.5754157	51.6226773
31	42.3794408	46.0002707	50.0026782	54.4294710
32	44.2270296	48.1502775	52.5027585	57.3345025
33	46.1115702	50.3540345	55.0778413	60.3412101
34	48.0338016	52.6128653	57.7301765	63.4531524
35	49.9944776	54.9282074	60.4620818	66.6740127
36	51.9943672	57.3014126	63.2759443	70.0076032
37	54.0342545	59.7339479	66.1742226	73.4578693
38	56.1149396	62.2272966	69.1594493	77.0288947
39	58.2372384	64.7829791	72.2342327	80.7249060
40	60.4019832	67.4025535	75.4012597	84.5502778
41	62.6100228	70.0876174	78.6632975	88.5095375
42	64.6822233	72.8398078	82.0231964	92.6073713
43	67.1594678	75.6608030	85.4838923	96.8486293
44	69.5026511	78.5523231	89.0484191	101.2383313
45	71.8927103	81.5161312	92.7198614	105.7816729
46	74.3305645	84.5540344	96.5014172	110.4840315
47	76.8171758	87.6678853	100.3965009	115.3509726
48	79.3535193	90.8595824	104.4083960	120.3882566
49	81.9405697	94.1310729	108.5406479	125.6018456
50	84.5794015	97.4843488	112.7968673	130.9979102

TABLE

TABLE IV.

The amount of 1 l. annuity, compound intereſt.

Y.	4 *per cent.*	$4\frac{1}{2}$ *per cent.*	5 *per cent.*	6 *per cent.*
1	1.0000000	1.0000000	1.0000000	1.0000000
2	2.0400000	2.0450000	2.0500000	2.0600000
3	3.1216000	3.1370250	3.1525000	3.1836000
4	4.2464640	4.2781911	4.3101250	4.3746016
5	5.4163226	5.4707097	5.5256312	5.6370930
6	6.6329755	6.7168917	6.8019128	6.9753187
7	7.8982945	8.0191518	8.1420084	8.3938378
8	9.2142263	9.3800136	9.5491089	9.8974681
9	10.5827953	10.8021142	11.0265643	11.4913162
10	12.0061071	12.2882094	12.5778925	13.1807958
11	13.4863514	13.8411788	14.2067871	14.9716435
12	15.0258055	15.4640318	15.9171265	16.8699420
13	16.6268377	17.1599133	17.7129828	18.8821385
14	18.2919112	18.9321094	19.5986320	21.0150667
15	20.0235876	20.7840543	21.5785636	23.2759707
16	21.8245311	22.7193367	23.6574918	25.6725289
17	23.6975124	24.7417069	25.8403664	28.2128806
18	25.6454129	26.8550837	28.1323847	30.9056534
19	27.6712294	29.0635625	30.5390039	33.7599925
20	29.7780786	31.3714228	33.0659541	36.7855920
21	31.9692017	33.7831368	35.7192518	39.9927275
22	34.2479698	36.3033779	38.5052144	43.3922911
23	36.6178886	38.9370299	41.4304751	46.9958285
24	39.0826041	41.6891963	44.5019989	50.8155782
25	41.6459083	44.5652101	47.7270988	54.8645128

TABLE IV.

The amount of 1 l. annuity, compound intereſt.

Y.	4 per cent.	$4\frac{1}{2}$ per cent.	5 per cent.	6 per cent.
26	44.3117446	47.5706446	51.1134538	59.1563827
27	47.0842144	50.7113236	54.6691265	63.7057657
28	49.9675830	53.9933332	58.4025828	68.5281116
29	52.9662863	57.4230332	62.3227119	73.6397983
30	56.0849377	61.0070698	66.4388475	69.0581862
31	59.3283352	64.7523878	70.7607899	84.8016774
32	62.7014687	68.6662452	75.2988294	90.8897780
33	66.2095274	72.7562263	80.0637708	97.3431647
34	69.8579085	77.0302565	85.0669594	104.1837546
35	73.6522248	81.4966180	90.3203073	111.4347799
36	77.5983138	86.1639658	95.8363227	119.1208667
37	81.7022464	91.0413443	101.6281388	127.2681187
38	85.9703362	96.1382048	107.7095458	135.9042058
39	90.4091497	101.4644240	114.0950231	145.0584581
40	95.0255157	107.0303231	120.7997742	154.7619656
41	99.8265363	112.8466876	127.8397829	165.0476836
42	104.8195978	118.9247885	135.2317511	175.9505446
43	110.0123817	125.2764040	142.9933386	187.5075772
44	115.4128169	131.9138422	151.1430056	199.7580319
45	121.0293920	138.8499651	159.7001559	212.7435138
46	126.8705677	146.0982135	168.6851637	226.5081246
47	132.9453904	153.6726331	178.1194218	241.0986121
48	139.2632060	161.5879016	188.0253929	256.5645288
49	145.8337342	169.8593572	198.4266626	272.9584006
50	152.6670836	178.5030282	209.3479957	290.3359046

TABLE

TABLE V.

The present worth of 1 l. annuity, compound interest.

Y.	2 *per cent.*	$2\frac{1}{2}$ *per cent.*	3 *per cent.*	$3\frac{1}{2}$ *per cent.*
1	0.9803922	0.9756098	0.9708738	0.9661836
2	1.9415609	1.9274242	1.9134697	1.8996943
3	2.8838833	2.8560236	2.8286114	2.8016371
4	3.8077287	3.7619742	3.7170984	3.6730792
5	4.7134595	4.6458285	4.5797072	4.5150524
6	5.6014309	5.5081254	5.4171914	5.3285530
7	6.4719911	6.3493906	6.2302829	6.1145439
8	7.3254814	7.1701372	7.0196922	6.8739555
9	8.1622367	7.9708655	7.7861089	7.6076865
10	8.9825850	8.7520639	8.5302028	8.3166053
11	9.7868480	9.5142087	9.2526241	9.0015510
12	10.5753412	10.2577646	9.9540040	9.6633343
13	11.3483737	10.9831839	10.6349553	10.3027385
14	12.1062487	11.6909122	11.2960731	10.9205203
15	12.8492635	12.3813777	11.9379351	11.5174109
16	13.5777093	13.0550027	12.5611020	12.0941168
17	14.2918719	13.7121977	13.1661185	12.6513206
18	14.9920313	14.3533636	13.7535131	13.1896817
19	15.6784620	14.9788913	14.3237991	13.7098374
20	16.3514333	15.5891623	14.8774748	14.2124033
21	17.0112092	16.1845486	15.4150241	14.6979742
22	17.6580482	16.7654132	15.9369166	15.1671248
23	18.2922041	17.3321105	16.4436084	15.6204105
24	18.9139256	17.8849858	16.9355421	16.0583676
25	19.5234565	18.4243764	17.4131477	16.4815146

TABLE V.

The present worth of 1 l. annuity, compound intereſt.

Y.	2 *per cent.*	$2\frac{1}{2}$ *per cent.*	3 *per cent.*	$3\frac{1}{2}$ *per cent.*
26	20.1210358	18.9506111	17.8768429	16.8903523
27	20.7068978	19.4640109	18.3270315	17.2853645
28	21.2812724	19.9648887	18.7641082	17.6670188
29	21.8443847	20.4535499	19.1884546	18.0357670
30	22.3964556	20.9302926	19.6004413	18.3920454
31	22.9377015	21.3954074	20.0004285	18.7362758
32	23.4683348	21.8491780	20.3887655	19.0688656
33	23.9885636	22.2918809	20.7657918	19.3902082
34	24.4985917	22.7237863	21.1318367	19.7006842
35	24.9986193	23.1451573	21.4872200	20.0006612
36	25.4888425	23.5562511	21.8322525	20.2904938
37	25.9694534	23.9573181	22.1672354	20.5705254
38	26.4406406	24.3486030	22.4924616	20.8410874
39	26.9025888	24.7303444	22.8082151	21.1024999
40	27.3554792	25.1027751	23.1147719	21.3550723
41	27.7994895	25.4661220	23.4123999	21.5991037
42	28.2347936	25.8206068	23.7013592	21.8348828
43	28.6615623	26.1664457	23.9819021	22.0626887
44	29.0799631	26.5038495	24.2542739	22.2827910
45	29.4901599	26.8330239	24.5187125	22.4954503
46	29.8[illegible]23[illegible]30	27.1541696	24.7754490	22.7009183
47	30.28658[illegible]0	27.4074826	25.0247078	22.8994378
48	30.6731196	27.7731537	25.2667066	23.0912443
49	31.0520780	28.0713695	25.5016569	23.2765645
50	31.4236059	28.3623117	25.7297640	23.4556179

TABLE V.

The present worth of 1 l. annuity, compound interest.

Y.	4 *per cent.*	4½ *per cent.*	5 *per cent.*	6 *per cent.*
1	0.9615385	0.9569378	0.9523809	0.9433962
2	1.8860947	1.8726678	1.8594104	1.8333926
3	2.7750910	2.7489644	2.7232480	2.6730119
4	3.6298952	3.5875257	3.5459505	3.4651056
5	4.4518223	4.3899767	4.3294767	4.2123638
6	5.2421369	5.1578725	5.0756921	4.9173244
7	6.6020547	5.8927009	5.7863734	5.5823815
8	6.7327448	6.5958861	6.4632128	6.2097939
9	7.4353314	7.2687905	7.1078217	6.8016923
10	8.1108955	7.9127182	7.7217349	7.3600871
11	8.7604763	8.5289169	8.3064142	7.8868747
12	9.3850733	9.1185808	8.8632516	8.3838440
13	9.9856473	9.6828524	9.3935730	8.8526831
14	10.5631223	10.2228253	9.8986409	9.2949840
15	11.1183868	10.7395457	10.3796580	9.7122491
16	11.6522949	11.2340151	10.8377695	10.1058953
17	12.1656680	11.7071914	11.2740662	10.4772597
18	12.6592961	12.1599918	11.6895869	10.8276035
19	13.1339385	12.5932936	12.0853208	11.1581165
20	13.5903253	13.0079365	12.4622103	11.4699213
21	14.0291589	13.4047239	12.8211527	11.7640767
22	14.4511142	13.7844248	13.1630026	12.0415818
23	14.8568405	14.1477749	13.4885739	12.3033790
24	15.2469619	14.4954784	13.7986418	12.5503576
25	15.6220787	14.8282089	14.0939445	12.7833562

TABLE V.

The present worth of 1 l. annuity, compound interest.

Y.	4 *per cent.*	$4\frac{1}{2}$ *per cent.*	5 *per cent.*	6 *per cent.*
26	15.9827678	15.1466115	14.3751853	13.0031663
27	16.3295844	15.4513028	14.6430336	13.2105342
28	16.6630618	15.7428735	14.8981272	13.4061644
29	16.9837132	16.0218885	15.1410735	13.5907211
30	17.2920318	16.2888885	15.3724510	13.7648312
31	17.5884921	16.5443909	15.5928104	13.9290861
32	17.8735500	16.7888909	15.8026766	14.0840435
33	18.1476441	17.0228621	16.0025491	14.2302297
34	18.4111962	17.2467580	16.1929039	14.3681412
35	18.6646116	17.4610124	16.3741942	14.4982465
36	18.9082803	17.6660406	16.5468516	14.6209872
37	19.1425771	17.8622398	16.7112872	14.7367804
38	19.3678625	18.0499902	16.8678926	14.8460192
39	19.5844831	18.2296557	17.0170406	14.9490747
40	19.7927721	18.4015844	17.1590862	15.0462969
41	19.9930500	18.5661095	17.2943678	15.1380160
42	20.1856250	18.7235498	17.4232074	15.2245434
43	20.3707931	18.8742103	17.5459118	15.3061730
44	20.5488395	19.0183831	17.6627732	15.3831821
45	20.7200378	19.1563474	17.7740697	15.4558321
46	20.8846517	19.2883707	17.8800663	15.5243699
47	21.0429342	19.4147088	17.9610155	15.5890282
48	21.1951289	19.5356066	18.0771576	15.6500266
49	21.3414700	19.6512981	18.1687215	15.7075723
50	21.4821826	19.7620078	18.2559253	15.7618610

TABLE

TABLE VI.

The annuity which 1 l. will purchase, compound interest.

Y.	2 per cent.	2½ per cent.	3 per cent.	3½ per cent.
1	1.0200000	1.0250000	1.0300000	1.0350000
2	.5150495	.5188272	.5226108	.5264005
3	.3467547	.3501372	.3535304	.3569342
4	.2626238	.2658179	.2690271	.2722511
5	.2121584	.2152469	.2183546	.2214814
6	.1785258	.1815499	.1845975	.1876682
7	.1545120	.1574954	.1605064	.1635445
8	.1365098	.1394674	.1424564	.1454767
9	.1225154	.1254569	.1284339	.1314460
10	.1113265	.1142588	.1172305	.1202414
11	.1021779	.1051060	.1080775	.1110920
12	.0945596	.0974871	.1004621	.1034840
13	.0881183	.0910483	.0940295	.0970616
14	.0826020	.0855365	.0885263	.0935707
15	.0778255	.0807665	.0837666	.0868251
16	.0736501	.0765990	.0796109	.0826848
17	.0699698	.0729278	.0759525	.0790431
18	.0667021	.0696701	.0727087	.0758168
19	.0637818	.0667606	.0698139	.0729403
20	.0611567	.0641471	.0672157	.0703610
21	.0587847	.0617873	.0648718	.0680366
22	.0566314	.0596466	.0627474	.0659321
23	.0546681	.0576964	.0608139	.0640188
24	.0528511	.0559128	.0590474	.0622728
25	.0512204	.0542759	.0574279	.0606740

TABLE

TABLE VI.

The annuity which 1 l. will purchase, compound interest.

Y.	2 per cent.	2½ per c.	3 per cent.	3½ per c.
26	.0496992	.0527688	.0559383	.0592054
27	.0482931	.0513769	.0545642	.0578524
28	.0469897	.0500879	.0532932	.0566027
29	.0457784	.0488913	.0521147	.0554454
30	.0446499	.0477776	.0510193	.0543713
31	.0435964	.0467490	.0499989	.0533724
32	.0426106	.0457683	.0490466	.0524415
33	.0416865	.0448591	.0481561	.0515724
34	.0408187	.0440068	.0473220	.0507597
35	.0400022	.0432056	.0465393	.0499984
36	.0392329	.0424516	.0458038	.0492842
37	.0385068	.0417409	.0451116	.0486133
38	.0378206	.0410701	.0444593	.0479821
39	.0371711	.0404362	.0438439	.0473878
40	.0365558	.0398362	.0432624	.0468273
41	.0359719	.0392679	.0427124	.0462982
42	.0354173	.0387288	.0421917	.0457983
43	.0348899	.0382169	.0416981	.0453254
44	.0343879	.0377304	.0412299	.0448777
45	.0339096	.0372675	.0407852	.0444534
46	.0334534	.0368268	.0403625	.0440511
47	.0330179	.0364067	.0399605	.0436692
48	.0326018	.0360060	.0395778	.0433065
49	.0322040	.0356235	.0392131	.0429617
50	.0318032	.0352581	.0388655	.0426337

TABLE

TABLE VI.

The annuity which 1 l. will purchase, compound interest.

Y.	4 *per c.*	4½ *per c.*	5 *per c.*	6 *per c.*
1	1.0400000	1.0450000	1.0500000	1.0600000
2	.5301961	.5339976	.5378049	.5454369
3	.3603485	.3637734	.3672086	.3741098
4	.2754901	.2787437	.2820118	.2885915
5	.2246271	.2277916	.2309748	.2373964
6	.1907619	.1938784	.1970157	.2033626
7	.1666096	.1097015	.1728198	.1791350
8	.1485279	.1516097	.1547218	.1610359
9	.1344930	.1375745	.1406901	.1470222
10	.1232909	.1263788	.1295046	.1358680
11	.1141490	.1172482	.1203890	.1207929
12	.1065522	.1096662	.1128254	.1192770
13	.1001437	.1032754	.1064558	.1129601
14	.0946690	.0978203	.1010240	.1275849
15	.0899411	.0931138	.0963423	.1029628
16	.0858200	.0890154	.0922699	.0989521
17	.0821985	.0854176	.0886991	.0954448
18	.0789933	.0822369	.0855462	.0923565
19	.0761386	.0794073	.0827450	.0896209
20	.0735818	.0768761	.0802426	.0871846
21	.0712801	.0740006	.0779961	.0850046
22	.0691988	.0725457	.0759705	.0830456
23	.0673091	.0706825	.0741368	.0812785
24	.0655868	.0689870	.0724709	.0796790
25	.0640121	.0674390	.0709525	.0782267

TABLE VI.

The annuity which 1 l. will purchase, compound interest.

Y.	4 *per c.*	$4\frac{1}{2}$ *per c.*	5 *per c.*	6 *per c.*
26	.0625674	.0660214	.0695643	.0769044
27	.0612385	.0647195	.0682919	.0756972
28	.0600130	.0635208	.0671225	.0745926
29	.0588799	.0624146	.0660455	.0735796
30	.0578301	.0613915	.0650514	.0726489
31	.0568554	.0604435	.0641321	.0717522
32	.0559486	.0595032	.0632804	.0710023
33	.0551036	.0587445	.0624900	.0702729
34	.0543148	.0579819	.0617554	.0695984
35	.0535773	.0572705	.0610717	.0689739
36	.0528869	.0566058	.0604345	.0683948
37	.0522396	.0559840	.0598398	.0678574
38	.0516319	.0554017	.0592842	.0673581
39	.0510608	.0548557	.0587640	.0668938
40	.0505235	.054343[illegible]	.0582782	.0664915
41	.0500174	.0538616	.0578223	.0660589
42	.0495402	.0534087	.0573947	.0656834
43	.0490899	.0529824	.0569933	.0653331
44	.0486645	.0525807	.0566163	.0650061
45	.0482625	.0522020	.0562617	.0647005
46	.0478821	.0518447	.0559282	.0644149
47	.0475219	.0515073	.0556142	.0641477
48	.0471807	.0511886	.0553184	.0638977
49	.0468571	.0508872	.0550397	.0636636
50	.0465502	.0506021	.0547767	.0634443

The

The use of Table IV.

1. Annuity, rate, and time, given, to find the amount.

Multiply the amount of 1 l. annuity, for the rate and time given, by the given annuity, and the product is the amount sought.

Ex. What will an annuity of 85 l. 10 s. amount to in 20 years, at 5 *per cent.* compound interest?

```
  33.0659541  amount of 1 l. annuity.
        85.5
 -----------
  1653297705
 1653297705
2645276328
------------
2827.13907555  amount sought.
```

2. Annuity, time, and amount, given, to find the rate.

Divide the amount by the annuity, and the quot will be the amount of 1 l.; which find in the table even with the given time, and on the head you have the rate.

Ex. At what rate *per cent.* will 85.5 amount to 2827.13907555?

```
85.5)2827.13907555
     -------------
      33.0659541   under 5 per cent.
```

3. Annuity, rate, and amount, given, to find the time.

Divide the amount by the annuity, and the quot will be the amount of 1 l.; which find under the given rate, and on the side you have the time.

Ex. In what time will an annuity of 85.5 l. amount to 2827.13907555 l. at 5 *per cent.*?

```
85.5)2827.13907555
     -------------
      33.0659541   opposite to 20 years.
```

 4. Amount

4. Amount, rate, and time, given, to find the annuity.

Divide the given amount by the amount of 1 l. for the rate and time given, and the quot is the annuity.

Ex. What annuity will amount to 2827.13907555 l. in 20 years, at 5 *per cent.* ?

33.0659541)2827.13907555
85.5 annuity.

The use of Table V.

1. Annuity, rate, and time, given, to find the present worth.

Multiply the present worth of 1 l. annuity for the rate and time given, by the given annuity, and the product is the present worth sought.

Ex. What is the present worth of an annuity of 100 l. to continue 15 years, discounting at 5 *per cent.* compound interest ?

L. s. d.
10.3796580 × 100 = 1037.9658 = 1037 19 $3\frac{3}{4}$. *Ans.*

2. Annuity, time, and present worth, given, to find the rate.

Divide the present worth by the annuity, and the quot is the present worth of 1 l. annuity; which find opposite to the time, and on the head you have the rate.

Ex. The present worth of an annuity of 100 l. to continue 15 years, is 1037.9658 l.: At what rate is interest computed ?

100)1037.9658(10.379658 found under 5 *per cent.*

3. Annuity, rate, and present worth, given, to find the time.

Divide the present worth by the annuity, and the quot is the present worth of 1 l. annuity; which find under the given rate, and you have the time on the side.

Ex. The present worth of an annuity of 100 l. discounting

counting at 5 *per cent.* is 1037.9658 l.: How many years will the annuity continue?

100)1037.9658(10.379658 found oppofite to 15 years.

4. Prefent worth, rate, and time, given, to find the annuity.

Divide the given prefent worth by the tabular number anfwering to the rate and time, and the quot is the annuity.

Ex. What annuity, to continue 15 years, will 1037.9658 l. purchafe, computing compound intereft at 5 *per cent.*?

10.379658)1037.965800(100 l. annuity.

The ufe of Table VI.

Price, or prefent worth, rate, and time, given, to find the annuity.

Multiply the price by the tabular number anfwering to the rate and time, and the product is the annuity fought.

Ex. What annuity, to continue 15 years, will 1037.9658 l. purchafe, reckoning at 5 *per cent.* compound intereft?

1037.9658 × .0963423 = 100 l. annuity.

Having thus fhown the ufe of each table feparately, or by itfelf, I fhall now fubjoin a few practical queftions; in folving feveral of which the promifcuous ufe of the tables will be neceffary.

Queft. 1. A pays 2000 l. for an annuity of 100 l. to continue 50 years; B puts 2000 l. out at intereft: Which of them will amount to the greateft fum at the end of the 50 years, at the rate of $4\frac{1}{2}$ *per cent.* compound intereft?

By

	L.
By Table I. the amount of 2000 l. for 50 years, at $4\frac{1}{2}$ *per cent.* is,	18065.27240
By Table IV. the amount of 100 l. annuity, for that time and rate, is,	17850.30282
B's amount exceeds that of A by	214.96958

By Table III. the present worth of 214.96958 l. is 23 l. 15 s. $11\frac{3}{4}$ d. discounting for 50 years at $4\frac{1}{2}$ *per cent.*; and so much was B's case better than that of A.

Quest. 2. A owes B 600 l. to be paid in 12 years, viz. 100 l. at the end of every 2 years; but agrees with B to pay him the whole in 6 years, by equal yearly payments, reckoning compound interest at 6 *per cent.*: What must the annual payment be?

By Table III. find the present worths of the 6 payments that were to be made in 12 years, and their sum will be 406.98272 l.

By Table VI. find what annuity for six years the said sum will purchase, viz. 406.98272 × .2033626 = 82.765 l. the annual payment.

Quest. 3. Which is most advantageous, a term of 15 years in an estate of 500 l. *per annum* or the reversion of the same estate for ever, after the expiration of the said 15 years, reckoning compound interest at 5 *per cent.*?

	L.
An estate of 500 l. *per annum* for ever, or, as it is usually expressed, in fee-simple, at 5 *per cent.* is worth a sum whose interest is 500 l. yearly, viz.	10000
By Table V. the present worth of the same estate, for a term of 15 years, at 5 *per cent.* is,	5189.829
And so the reversion is worth	4810.171

From the value of the term, viz. -	5189.829
Subtract the value of the reversion, -	4810.171
The term is better than the reversion by	379.658

Quest.

Quest. 4. A has a term of 7 years in an estate of 100 l. *per annum*; B has a term of 14 years in the same estate in reversion, after the expiration of the 7 years; and C has a further term of 21 years in reversion, after A and B: Required the present worths of the several terms, discounting at $4\frac{1}{2}$ *per cent.* compound interest.

By Table V. find the present worth of the yearly rent or annuity, for 42 years, for 21 years, for 7 years; subtract these present worths from each other, and the remainders are the answers, as under.

			L.	s.	d.
42 years,	18.7235498 × 100 =	1872.35498 =	1872	7	1
21 years,	13.4047239 × 100 =	1340.47239 =	1340	9	$5\frac{1}{4}$
7 years,	5.8927009 × 100 =	589.27009 =	589	5	$4\frac{3}{4}$

L.	s.	d.
589	5	$4\frac{3}{4}$
A's term.		

L.	s.	d.
1340	9	$5\frac{1}{4}$
589	5	$4\frac{3}{4}$
751	4	$0\frac{1}{2}$
B's term.		

L.	s.	d.
1872	7	1
1340	9	$5\frac{1}{4}$
531	17	$7\frac{3}{4}$
C's term.		

Quest. 5. A person having 9 years to run, in a lease for 49 years, of an estate of 80 l. *per annum*, would know what sum he ought to pay presently, in order to have the lease renewed or completed, by having 40 years added thereto, reckoning at 6 *per cent.* compound interest?

	L.
By Table V. the present worth of 1 l. annuity, at 6 *per cent.* for 49 years, is	15.7075723
By the same table, the value of 1 l. annuity, at 6 *per cent.* for 9 years, is	6.8016923
Their difference is	8.9058800
Which multiplied by	80
Gives for answer,	712.4704

Quest. 6. A farmer gets a lease of 21 years renewed, for a fine or grassum of 400 l. and it is put in his option, either

either to pay the fine presently, or to pay yearly an additional rent equivalent thereto: Required the additional rent, reckoning compound interest at 5 *per cent.*

By Tab. VI. the annuity for 21 years that 1 l. will purchase, at 5 *per cent.* is	.0779961
Which multiplied by	400
Gives the additional rent	31.1984400

Quest. 7. What annuity, to continue 19 years, and to commence presently, may be purchased by a bill of 2000 l. payable 3 years hence, reckoning at 6 *per cent.* compound interest?

	L.
By Table III. the present worth of 2000 l. due 3 years hence, at 6 *per cent.* is	1679.238600
By Table VI. the annuity which 1679.2386 l. will purchase for 19 years, at 6 *per cent.* is -	150.494874

Quest. 8. What annuity would be sufficient to pay off a debt of 120 millions, in the space of 30 years, at 4 *per cent.* compound interest?

By Table VI. the annuity for 30 years that 1 l. will purchase, at 4 *per cent.* is	.0578301
Which multiply by the debt	120000000
The product is the annuity sought, viz.	L. 6939612
From which subtract the yearly interest of 120 millions, at 4 *per cent.* viz.	4800000
The remainder, if we suppose the 120 millions to be the national debt, will be a sinking fund sufficient to clear the whole in 30 years. - -	L. 2139612

From the above example, appears the conveniency, or rather the necessity, of continuing the decimals in the tables to seven places at least.

Quest. 9. For a certain lease, to continue 7 years, A offers 650 l. fine, and 200 l. *per annum*; B offers 150 l. fine,

fine, and 300 l. *per annum*; and C offers 1800 l. fine, without any yearly rent: Which of these three offers is the best, reckoning at 5 *per cent.* compound interest?

	L.
By Table V. the present worth of 200 l. *per annum* for 7 years, at 5 *per c.* is	1157.27468
To which add the fine	650
Value of A's offer	1807.27468
By Table V. the present worth of 300 l. *per annum* for 7 years, at 5 *per c.* is	1735.91202
To which add the fine	150
Value of B's offer	1885.91202
C's offer is	1800

By which it appears that B's offer is the best; and that, neglecting the decimal, he offers 78 l. more than A, and 85 l. more than C.

Quest. 10. A lease of an estate, to continue 14 years, is offered for 250 l. fine, and 44 l. yearly rent; but the tenant wants to reduce the rent to 20 l. *per annum*: What ought the fine to be, reckoning compound interest at 6 *per cent.*?

The difference between 44 and 20, the two rents, is 24 l.

	L.
By Table V. the present worth of 24 l. for 14 years, at 6 *per cent.* is	223.0796160
To which add the first fine	250
The fine sought	473.0796160

II. *Annuities for ever, or freehold estates.*

In freehold estates, commonly called *annuities in fee-simple*, the things chiefly to be considered are, 1. The annuity or yearly rent. 2. The price or present worth. 3. The rate of interest. The questions that usually occur on this head will fall under one or other of the following problems.

Prob. 1. Annuity and rate of intereſt given, to find the price.

As the rate of 1 l. to 1 l. ſo the rent to the price.

Ex. The yearly rent of a ſmall eſtate is 40 l.: What is it worth in ready money, computing intereſt at $3\frac{1}{2}$ *per cent.*?

	L.	s.	d.
As .035 : 1 :: 40 : 1142.857142 =	1142	17	$1\frac{1}{2}$

Prob. 2. Price and rate of intereſt given, to find the rent or annuity.

As 1 l. to its rate, ſo the price to the rent.

Ex. A gentleman purchaſes an eſtate for 4000 l. and has $4\frac{1}{2}$ *per cent.* for his money: Required the rent.

As 1 : .045 :: 4000 : 180 l. rent ſought.

Prob. 3. Price and rent given, to find the rate of intereſt.

As the price to the rent, ſo 1 to the rate.

Ex. An eſtate of 180 l. yearly rent is bought for 4000 l.: What rate of intereſt has the purchaſer for his money?

As 4000 : 180 :: 1 : .045 rate ſought.

Prob. 4. The rate of intereſt given, to find how many years purchaſe an eſtate is worth.

Divide 1 by the rate, and the quot is the number of years purchaſe the eſtate is worth.

Ex. A gentleman is willing to purchaſe an eſtate, provided he can have $2\frac{1}{2}$ *per cent.* for his money: How many years purchaſe may he offer?

.025)1.000(40 years purchaſe. *Anſ.*

Prob. 5. The number of years purchaſe at which an eſtate is bought or ſold, given, to find the rate of intereſt.

Divide

Divide 1 by the number of years purchaſe, and the quot is the rate of intereſt.

Ex. A gentleman gives 40 years purchaſe for an eſtate: What intereſt has he for his money?

40)1.000(.025 rate ſought.

The computations hitherto are all performed by a ſingle diviſion or multiplication, and the learner will ſcarcely perceive that the operations are conducted by the rules of compound intereſt; but when a reverſion occurs, there will be occaſion for the tables, as follows.

Prob. 6. The rate of intereſt, and the rent of a freehold eſtate in reverſion, given, to find the preſent worth or value of the reverſion.

By Prob. 1. find the price or preſent worth of the eſtate, as if poſſeſſion was to commence preſently; and then, by Table V. find the preſent value of the given annuity, or rent, for the years prior to the commencement, ſubtract this value from the former value, and the remainder is the value of the reverſion.

Ex. A has the poſſeſſion of an eſtate of 130 l. *per annum*, to continue 20 years; B has the reverſion of the ſame eſtate from that time for ever: What is the value of the eſtate, what the value of the 20 years poſſeſſion, and what the value of the reverſion, reckoning compound intereſt at 6 *per cent.*?

By Prob. 1. .06)130.00(2166.6666 value of the eſtate.
By Table V. 1491.0896 value of the poſſeſſion.
675.5770 value of the reverſion.

Prob. 7. The price or value of a reverſion, the time prior to the commencement, and rate of intereſt, given, to find the annuity or rent.

By Table I. find the amount of the price of the reverſion for the years prior to the commencement; and

 then,

then, py Prob. 2. find the annuity which that amount will purchaſe.

Ex. The reverſion of a freehold-eſtate, to commence 20 years hence, is bought for 675.577 l. compound intereſt being allowed at 6 *per cent.*: Required the annuity or rent.

By Table I. the amount of 675.577 l. for 20 years, at 6 *per cent.* is } L. 2166.8

By Prob. 2. 2166.8 × .06 = 130.0 rent ſought

III. *Life Annuities.*

The value of annuities for life is determined from obſervations made on the bills of mortality. Dr Halley, Mr Simpſon, and Monſ. de Moivre, are gentlemen of diſtinguiſhed merit in calculations of this kind.

Dr Halley had recourſe to the bills of mortality at Breſlaw, the capital of Sileſia, as a proper ſtandard for the other parts of Europe, being a place pretty central, at a diſtance from the ſea, and not much crouded with traffickers or foreigners. He pitches upon 1000 perſons all born in one year, and obſerves how many of theſe were alive every year, from their birth to the extinction of the laſt, and conſequently how many died each year, as in the following table.

Dr

Dr Halley's table on the bills of mortality at Breslaw.

Age.	Perf. liv.	A.	Perf. liv.	A.	Perf. liv.	A.	Perf. liv.
1	1000	24	573	47	377	70	142
2	855	25	567	48	367	71	131
3	798	26	560	49	357	72	120
4	760	27	553	50	346	73	109
5	732	28	546	51	335	74	98
6	710	29	539	52	324	75	88
7	692	30	531	53	313	76	78
8	680	31	523	54	302	77	68
9	670	32	515	55	292	78	58
10	661	33	507	56	282	79	49
11	653	34	499	57	272	80	41
12	646	35	490	58	262	81	34
13	640	36	481	59	252	82	28
14	634	37	472	60	242	83	23
15	628	38	463	61	232	84	20
16	622	39	454	62	222	85	15
17	616	40	445	63	212	86	11
18	610	41	436	64	202	87	8
19	604	42	427	65	192	88	5
20	598	43	417	66	182	89	3
21	592	44	407	67	172	90	1
22	586	45	397	68	162	91	0
23	579	46	387	69	152		

The above table is well adapted to Europe in general; but in the city of London, there is obferved to be a greater difparity in the births and burials than in any other place, owing probably to the vaft refort of people thither, in the way of commerce, from all parts of the known world. Mr Simpfon, therefore, in order to have a table particularly fuited to this populous city, pitches upon 1280 perfons, all born the fame year, and records

records the number remaining alive each year, till none were in life; as in the following table.

Mr Simpson's table on the bills of mortality at London.

Age.	Perf. liv.	A.	Perf. liv.	A.	Perf. liv.	A.	Perf. liv.
0	1280	24	434	48	220	72	59
1	870	25	426	49	212	73	54
2	700	26	418	50	204	74	49
3	635	27	410	51	196	75	45
4	600	28	402	52	188	76	41
5	580	29	394	53	180	77	38
6	564	30	385	54	172	78	35
7	551	31	376	55	165	79	32
8	541	32	367	56	158	80	29
9	532	33	358	57	151	81	26
10	524	34	349	58	144	82	23
11	517	35	340	59	137	83	20
12	510	36	331	60	130	84	17
13	504	37	322	61	123	85	14
14	498	38	313	62	117	86	12
15	492	39	304	63	111	87	10
16	486	40	294	64	105	88	8
17	480	41	284	65	99	89	6
18	474	42	274	66	93	90	5
19	468	43	264	67	87	91	4
20	462	44	255	68	81	92	3
21	455	45	246	69	75	93	2
22	448	46	237	70	69	94	1
23	441	47	228	71	64	95	0

It may not be improper in this place to obferve, that however perfect tables of this fort may be in themfelves, and however well adapted to any particular climate, yet the conclufions deduced from them muft always be uncertain, being nothing more than probabilities, or conjectures drawn from the ufual period of human life. And the modifh practice of buying and felling annuities on

on lives, by rules founded on ſuch principles, may be juſtly conſidered as a ſort of lottery or chance-work, in which the parties concerned muſt often be deceived. But as eſtimates and computations of this kind are now become faſhionable, I ſhall here give ſome brief account of ſuch as appear moſt material.

From the above tables, the probability of the continuance or extinction of human life is eſtimated as follows.

1. The probability that a perſon of a given age ſhall live a certain number of years, is meaſured by the proportion which the number of perſons living at the propoſed age has to the difference between the ſaid number and the number of perſons living at the given age.

Thus, if it be demanded, what chance a perſon of 40 years has to live 7 years longer? from 445, the number of perſons living at 40 years of age in Dr Halley's table, ſubtract 377, the number of perſons living at 47 years of age, and the remainder, 68, is the number of perſons that died during theſe 7 years; and the probability or chance that the perſon in the queſtion ſhall live theſe 7 years is as 377 to 68, or nearly as $5\frac{1}{2}$ to 1. But by Mr Simpſon's table, the chance is ſomething leſs than that of 4 to 1.

2. If the year to which a perſon of a given age has an equal chance of arriving before he dies, be required, it may be found thus: Find half the number of perſons living at the given age in the tables, and in the column of age you have the year required.

Thus, if the queſtion be put with reſpect to a perſon of 30 years of age, the number of that age in Dr Halley's table is 531, the half whereof is 265, which is found in the table between 57 and 58 years; ſo that a perſon of 30 years has an equal chance of living between 27 and 28 years longer.

3. By the tables, the premium of inſurance upon lives may in ſome meaſure be regulated.

Thus, the chance that a perſon of 25 years has to live another year, is, by Dr Halley's table, as 80 to 1; but the chance that a perſon of 50 years has to live a year

year longer is only 30 to 1. And, consequently, the premium for insuring the former ought to be the premium for insuring the latter for one year, as 30 to 80, or as 3 to 8.

4. By the tables may be computed the value of an annuity for life.

Thus, if it be required to find the value of an annuity of 1 l. for the life of a person of 30 years of age, interest 5 *per cent.* by Table V. in *Annuities certain*, find the value of 1 l. annuity for 1 year, at 5 *per cent.* viz. .952. Now, it is obvious, that this .952 would be the true value of the first year's annuity, provided the purchaser was sure of receiving payment: but the person may die; and therefore this value must be lessened in proportion to the risk, by the following analogy from Dr Halley's table, viz. as 531 to 523, so .952 to .9375, the true value of the first year's annuity. In like manner may the value of the second year's annuity be found: for, by Table V. the value of 1 l. annuity for 2 years, at 5 *per cent.* is 1.8594; from which deduce the value of 1 year's annuity, viz. .9523, and there remains for the separate value of the second year's annuity .9071. Then, from the table. say, As 531 to 515, so .9071 to .8797, the true value of the second year's annuity. And by proceeding in this manner, the separate value of every year's annuity, to the utmost extent of life, may be found; the sum of all which will be the value of the annuity sought.

But the above method being tedious, and therefore unfit for practice, I shall here lay down another method, much shorter and easier, and shall deliver what seems necessary to be said on life-annuities in the way of problems. Previous to which, it will be proper to observe, that in calculations of this kind, the age of 86 years is esteemed the utmost extent or limit of life; and the difference between the given age and 86 is called the *complement of life*.

PROB.

PROB. I.

To find the value of an annuity of 1 l. for the life of a single person of any given age.

Monf. de Moivre, by observing the decrease of the probabilities of life, as exhibited in the table, composed an algebraic theorem or canon, for computing the value of an annuity for life; which canon I shall here lay down by way of

RULE.

Find the complement of life; and, by Table V. in *Annuities Certain*, find the value of 1 l annuity for the years denoted by the said complement; multiply this value by the amount of 1 l. for a year, and divide the product by the complement of life; then subtract the quot from 1; divide the remainder by the interest of 1 l. for a year; and this last quot will be the value of the annuity sought; or, in other words, the number of years purchase the annuity is worth.

Ex. What is the value of an annuity of 1 l. for an age of 50 years, interest at 5 *per cent.*?

86
50 age given.
36 complement of life.

By Table V. the value is 16.5468
Amount of 1 l. for a year, 1.05

827340
165468

Complement of life, 36)17.374140(.482615

From unity, viz. 1.000000
Subtract .482615

Interest of 1 l. .05).517385(10.3477 value sought.

By the preceding problem is constructed the following table.

The value of 1 l. annuity for a single life.

Age.	3 *per c.*	$3\frac{1}{2}$ *per c.*	4 *per c.*	$4\frac{1}{2}$ *per c.*	5 *per c.*	6 *per c.*
9 = 10	19.87	18.27	16.88	15.67	14.60	12.80
8 = 11	19.74	18.16	16.79	15.59	14.53	12.75
7 = 12	19.60	18.05	16.64	15.51	14.47	12.70
13	19.47	17.94	16.60	15.43	14.41	12.65
6 = 14	19.33	17.82	16.50	15.35	14.34	12.60
15	19.19	17.71	16.41	15.27	14.27	12.55
16	19.05	17.59	16.31	15.19	14.20	12.50
5 = 17	18.90	17.46	16.21	15.10	14.12	12.45
18	18.76	17.33	16.10	15.01	14.05	12.40
19	18.61	17.21	15.99	14.92	13.97	13.35
4 = 20	18.46	17.09	15.89	14.83	13.89	12.30
21	18.30	16.96	15.78	14.73	13.81	12.20
22	18.15	16.83	15.67	14.64	13.72	12.15
23	17.99	16.69	15.55	14.54	13.64	12.10
3 = 24	17.83	16.56	15.43	14.44	13.55	12.00
25	17.66	16.42	15.31	14.34	13.46	11.95
26	17.50	16.28	15.19	14.23	13.37	11.90
27	17.33	16.13	15.04	14.12	13.28	11.80
28	17.16	15.98	14.94	14.02	13.18	11.75
29	16.98	15.83	14.81	13.90	13.09	11.65
30	16.80	15.68	14.68	13.79	12.99	11.60
2 = 31	16.62	15.53	14.54	13.67	12.88	11.50
32	16.44	15.37	14.41	13.55	12.78	11.40
33	16.25	15.21	14.27	13.43	12.67	11.35
34	16.06	15.05	14.12	13.30	12.56	11.25
35	15.86	14.89	13.98	13.17	12.45	11.15
36	15.67	14.71	13.82	13.04	12.33	11.05
37	15.46	14.52	13.67	12.90	12.21	11.00
38	15.29	14.34	13.52	12.77	12.09	10.90
1 = 39	15.05	14.16	13.36	12.63	11.96	10.80
40	14.84	13.98	13.20	12.48	11.83	10.70

The value of 1 l. annuity for a single life.

A	3 per c.	$3\frac{1}{2}$ per c	4 per c.	$4\frac{1}{2}$ per c	5 per c.	6 per c.
41	14.63	13.79	13.02	12.33	11.70	10.55
42	14.41	13.59	12.85	12.18	11.57	10.45
43	14.19	13.40	12.68	12.02	11.43	10.35
44	13.96	13.20	12.50	11.87	11.29	10.25
45	13.73	12.99	12.32	11.70	11.14	10.10
46	13.49	12.78	12.13	11.54	10.99	10.00
47	13.25	12.56	11.94	11.37	10.84	9.85
48	13.01	12.36	11.74	11.19	10.68	9.75
49	12.76	12.14	11.54	11.00	10.51	9.60
50	12.51	11.92	11.34	10.82	10.35	9.45
51	12.26	11.69	11.13	10.64	10.17	9.30
52	12.00	11.45	10.92	10.44	9.99	9.20
53	11.73	11.20	10.70	10.24	9.82	9.00
54	11.46	10.95	10.47	10.04	9.63	8.85
55	11.18	10.69	10.24	9.82	9.44	8.70
56	10.90	10.44	10.01	9.61	9.24	8.55
57	10.61	10.18	9.77	9.39	9.04	8.35
58	10.32	9.91	9.52	9.16	8.83	8.20
59	10.03	9.64	9.27	8.93	8.61	8.00
60	9.73	9.36	9.01	8.69	8.39	7.80
61	9.42	9.08	8.75	8.44	8.16	7.60
62	9.11	8.79	8.48	8.19	7.93	7.40
63	8.79	8.49	8.20	7.94	7.68	7.20
64	8.46	8.19	7.92	7.67	7.43	6.95
65	8.13	7.88	7.63	7.39	7.18	6.75
66	7.79	7.56	7.33	7.12	6.91	6.50
67	7.45	7.24	7.02	6.83	6.64	6.25
68	7.10	6.91	6.75	6.54	6.36	6.00
69	6.75	6.57	6.39	6.23	6.07	5.75
70	6.38	6.22	6.06	5.92	5.77	5.50

The value of 1 l. annuity for a single life.

A.	3 per c.	$3\frac{1}{2}$ per c.	4 per c.	$4\frac{1}{2}$ per c	5 per c.	6 per c.
71	6.01	5.87	5.72	5.59	5.47	5.20
72	5.63	5.51	5.38	5.26	5.15	4.90
73	5.25	5.14	5.02	4.92	4.82	4.60
74	4.85	4.77	4.66	4.57	4.49	4.30
75	4.45	4.38	4.29	4.22	4.14	4.00
76	4.05	3.98	3.91	3.84	3.78	3.65
77	3.63	3.57	3.52	3.47	3.41	3.30
78	3.21	3.16	3.11	3.07	3.03	2.95
79	2.78	2.74	2.70	2.67	2.64	2.55
80	2.34	2.31	2.28	2.26	2.23	2.15

The above table shews the value of an annuity of one pound for a single life, at all the current rates of interest; and is esteemed the best table of this kind extant, and preferable to any other of a different construction. But yet those who sell annuities have generally one and a half or two years more value than specified in the table, from purchasers whose age is 20 years or upwards.

Annuities of this sort are commonly bought or sold at so many years purchase; and the value assigned in the table may be so reckoned. Thus the value of an annuity of one pound for an age of 50 years, at 3 *per cent.* interest, is 12.51; that is, 12 l. 10 s. or twelve and a half years purchase. The marginal figures on the left of the column of age serve to shorten the table, and signify, that the value of an annuity for the age denoted by them, is the same with the value of an annuity for the age denoted by the numbers before which they stand. Thus the value of an annuity for the age of 9 and 10 years is the same; and the value of an annuity for the age of 6 and 14, for the age of 3 and 24, &c. is the same. The further use of the table will appear in the questions and problems following.

Quest.

Quest. 1. A perſon of 50 years would purchaſe an annuity for life of 200 l.: What ready money ought he to pay, reckoning intereſt at $4\frac{1}{2}$ *per cent.*?

	L.
By the table the value of 1 l. is	10.82
Multiply by	200
Value to be paid in ready money	2164.00 *Anſ.*

Queſt. 2. A young merchant marries a widow lady of 40 years of age, with a jointure of 300 l. a-year, and wants to diſpoſe of the jointure for ready money: What ſum ought he to receive, reckoning intereſt at $3\frac{1}{2}$ *per cent.*?

	L.
By the table the value of 1 l. is	13.98
	300
Value to be received in ready money	4194.00 *Anſ.*

If an annuity for life, together with a reverſion for a term of years, or the reverſion by itſelf, is offered; in this caſe reduce the years purchaſe found in the table, to years certain, by Table V.; and to the years certain add the years in reverſion; and the preſent worth correſponding to the ſum of the years in Table V. will be the value of the life and reverſion; from which, if the value of the life be ſubtracted, there will remain the value of the reverſion.

Queſt. 3. What is the preſent worth or value of an eſtate of 80 l. yearly rent, together with a reverſion of 20 years after the death of the preſent poſſeſſor, ſuppoſed to be 40 years of age? and what is the value of the reverſion by itſelf, reckoning intereſt at 5 *per cent.*?

The value of the life by the above table is 11.83; and the neareſt value to this in Table V. under 5 *per cent.* is is 11.6895869; oppoſite to which is 18 years, and 18 + 20 = 38 years; and by Table V. the value of 1 l. annuity for 38 years at 5 *per cent.* is 16.8678926; which multiplied

multiplied by 80 gives 1349.4314080 l. for the value of the life and reversion.

To find the value of the reversion.

From	16.8678926
Subtract	11.6895869
Value of 1 l. reversion	5.1783057
Multiply by	80
Value of 80 l. reversion	414.2644560

If the value of a reversion in fee-simple, or for ever, after a life of a given age, be required; from the value of the fee-simple, or perpetuity, subtract the value of the life in possession; and the remainder will be the value of the reversion.

Quest. 4. A widow-lady of 60 years of age is in possession for life of an estate of L. 500 *per annum*: What is the value of the reversion in fee-simple, interest at 5 *per cent.*?

		L.
Value of 1 l. in fee, is	- -	20
Value of the life by the table is	-	8.39
Value of 1 l. in reversion,	- -	11.61
	Multiply by	500
Value of 500 l. in reversion	- -	5805.00

If the reversion depends on two joint lives, or on the longest of two lives, find the value of the joint lives, or of the longest of the two lives, by Prob. II. or Prob. III. following, and subtract the value so found from the value of the perpetuity, or estate in fee-simple.

PROB. II.

To find the value of an annuity for the joint continuance of two lives, one life failing, the annuity to cease.

Here there are two cases, according as the ages of the two persons are equal or unequal.

1.

1. If the two perſons be of the ſame age, work by the following

RULE.

Take the value of any one of the lives from the table, multiply this value by the intereſt of 1 l. for a year, ſubtract the product from 2, divide the foreſaid value by the remainder, and the quot will be the value of 1 l. annuity, or the number of years purchaſe ſought.

Ex. What is the value of 100 l. annuity for the joint lives of two perſons, of the age of 30 years each, reckoning intereſt at 4 *per cent.*?

By the table, one life of 30 years is	-	14.68
Multiply by	-	.04
Subtract the product		.5872
From	-	2.0000
Remains	-	1.4128

And 1.4128)14.68(10.39 value of 1 l. annuity.

And 10.39 × 100 = 1039 the value ſought.

2 If the two perſons are of different ages, work as directed in the following

RULE.

Take the values of the two lives from the table, multiply them into one another, calling the reſult the firſt product; then multiply the ſaid firſt product by the intereſt of 1 l. for a year, calling the reſult the ſecond product; add the values of the two lives, and from their ſum ſubtract the ſecond product; divide the firſt product by the remainder, and the quot will be the value of 1 l. annuity, or the number of years purchaſe ſought.

Ex. What is the value of 70 l. annuity for the joint lives of two perſons, whereof one is 40, and the other 50 years of age, reckoning intereſt at 5 *per cent.*?

By

By the table, the value of 40 years is	-	11.83
And the value of 50 years is	- -	10.35
First product,		122.4405
Multiply by	-	.05
Second product,		6.122025
Sum of the two lives,	- -	22.180000
Second product deduct,	-	6.122025
Remainder,	-	16.057975

And 16.057975)122.4405(7.62 value of 1 l. annuity.
70
533.40 value sought.

PROB. III.

To find the value of an annuity upon the longest of two lives; that is, to continue so long as either of the persons is in life.

RULE.

From the sum of the values of the single lives, subtract the value of the joint lives, and the remainder will be the value sought.

Ex. What is the value of an annuity of 1 l. upon the longest of two lives, the one person being 30, and the other 40 years of age, interest at 4 *per cent.*?

By the table, 30 years is	- -	14.68
40 years is	- - -	13.20
		27.88
Value of their joint lives, by Prob. II. Case 2. is	- - -	9.62
Value sought,	- - -	18.26

If the annuity be any other than 1 l. multiply the answer found as above by the given annuity.

If the two persons be of equal age, find the value of their joint lives by Case 1. of Prob. II.

PROB.

PROB. IV.

To find the value of the next preſentation to a living.

RULE.

From the value of the ſucceſſor's life, ſubtract the joint value of his and the incumbent's life, and the remainder will be the value of 1 l. annuity; which multiplied by the yearly income, will give the ſum to be paid for the next preſentation.

Ex. A enjoys a living of 100 l. *per annum*, and B would purchaſe the ſaid living for his life after A's death: The queſtion is, What he ought to pay for it, reckoning intereſt at 5 *per cent*. A being 60, and B 25 years of age?

			L.
By the table, B's life is	-	-	13.46
Joint value of both lives, by Prob. II. is			6.97
The value of 1 l. annuity,	-	-	6.49
Multiply by	-	-	100
Value of next preſentation,	-	-	649.00

The value of a direct preſentation is the ſame as that of any other annuity for life, and is found for 1 l. by the table; which being multiplied by the yearly income, gives the value ſought.

PROB. V.

To find the value of a reverſion for ever, after two ſucceſſive lives; or to find the value of a living after the death of the preſent incumbent and his ſucceſſor.

RULE.

By Prob. III. find the value of the longeſt of the two lives, and ſubtract that value from the value of the perpetuity, and the remainder will be the value ſought.

Ex. A, aged 50, enjoys an eſtate or living of 100 l. *per annum*; B, aged 30, is intitled to his lifetime of the

ſame eſtate after A's death; and it is propoſed to ſell the eſtate juſt now, with the burden of A and B's lives on it: What is the reverſion worth, reckoning intereſt at 4 *per cent.*?

		L.
By the table, A's life of 50 is	-	11.34
B's life of 30 is	-	14.68
	Sum,	26.02
Value of their joint lives, found by Prob. II. Caſe 2. is	-	8.60
Value of the longeſt life,	-	17.42 ſub.
From the value of the perpetuity,	-	25.00
Remains the value of 1 l. reverſion,	-	7.58
	Multiply by	100
Value of the reverſion,	-	758.00

PROB. VI.

To find the value of the joint continuance of three lives, one life failing, the annuity to ceaſe.

RULE.

Find the ſingle values of the three lives from the table; multiply theſe ſingle values continually, calling the reſult the product of the three lives; multiply that product by the intereſt of 1 l. and that product again by 2, calling the reſult the double product; then, from the ſum of the ſeveral products of the lives taken two and two, ſubtract the double product; divide the product of the three lives by the remainder, and the quot will be the value of the three joint lives.

Ex. A is 18 years of age, B 34, and C 56: What is the value of their joint lives, reckoning intereſt at 4 *per cent.*?

By the table, the value of A's life is 16.1, of B's 14.12, and of C's 10.01.

$16.1 \times 14.12 \times 10.01 = 2275.6$ product of the 3 lives.

$$\begin{array}{r} .04 \\ \hline 91.024 \\ 2 \\ \hline 182.048 \end{array}$$ double product.

Product of A and B,	16.1	× 14.12	= 227.33
A and C,	16.1	× 10.01	= 161.16
B and C,	14.12	× 10.01	= 141.34
Sum of all, two and two,		-	529.83
Double product subtract,		-	182.048
Remainder,		-	347.782

And 347.782)2275.600(6.54 value sought.

PROB. VII.

To find the value of an annuity upon the longest of three lives.

RULE.

From the sum of the values of the three single lives taken from the table, subtract the sum of all the joint lives taken two and two, as found by Prob. II. and to the remainder add the value of the three joint lives, as found by Prob. VI. and that sum will be the value of the longest life sought.

Ex. A is 18 years of age, B 34, and C 56: What is the value of the longest of these three lives, interest at 4 *per cent.*?

By the table, the single value of A's life is	16.1
single value of B's life is	14.12
single value of C's life is	10.01
Sum of the single values,	40.23
By Prob. II. the joint value of A and B is	10.76
joint value of A and C is	8.19
joint value of B and C is	7.65
Sum of the joint lives,	26.60
Remainder, - -	13.63
By Prob. VI. the value of the 3 joint lives is	6.54
Value of the longest of the three lives,	20.17

Other problems might be added, but these adduced are the more useful, and sufficient for most purposes. The reader probably may wish, that the reason of the rules, which, it must be owned, are intricate, had been assigned; but this could not be done without entering deeper into the subject than was practicable in this place. They who want further satisfaction on this head may have recourse to Mons. de Moivre's Doctrine of Chances, and other authors on this subject.

I shall now conclude *Life-annuities* by presenting the reader with a table, shewing how many years of an annuity, or annual income, will reimburse the purchaser or annuitant of the price or sum paid for the annuity, with the compound interest arising from the price as a capital or principal sum; that is, the table shews how many years and days possession will indemnify the purchaser, or save him from being a loser.

The

The possession that will reimburse the annuitant.

Years purch.	3 per c. Y.	da.	3½ per c. Y.	da.	4 per c. Y.	da.	4½ per c. Y.	da.	5 per c. Y.	da.	6 per c. Y.	da.
5	5	182	5	216	5	252	5	289	5	327	6	44
5½	6	37	6	79	6	122	6	168	6	216	6	319
6	6	261	6	311	6	364	7	55	7	113	7	241
6½	7	124	7	184	7	247	7	314	8	20	8	176
7	7	356	8	62	8	137	8	217	8	303	9	127
7½	8	227	8	311	9	34	9	129	9	231	10	95
8	9	104	9	200	9	304	10	51	10	172	11	81
8½	9	350	10	97	10	217	10	348	11	125	12	88
9	10	236	11		11	138	11	290	12	92	13	119
9½	11	128	11	274	12	69	12	245	13	75	14	177
10	12	24	12	191	13	9	13	212	14	75	15	265
10½	12	292	13	115	13	324	14	194	15	94	17	23
11	13	200	14	48	14	286	15	190	16	134	18	188
11½	14	115	14	354	15	259	16	203	17	196	20	36
12	15	36	15	305	16	246	17	234	18	285	21	309
12½	15	329	16	265	17	246	18	285	20	38	23	289
13	16	264	17	235	18	261	19	358	21	189	25	360
13½	17	206	18	216	19	292	21	90	23	13	28	183
14	18	156	19	209	20	340	22	215	24	247	31	164
14½	19	115	20	215	22	43	24	5	26	168	35	5
15	20	82	21	234	23	132	25	195	28	151	39	189
15½	21	59	22	267	24	245	27	60	30	209	45	233
16	22	45	23	316	26	18	28	336	32	360	55	88
16½	23	41	25	16	27	185	30	300	35	264	79	12

The

The use of the Table.

The left-hand column contains the number of years purchase paid for the annuity, or the number of pounds paid in ready money for each pound of the annuity; and the other columns show how long the annuitant must be in possession before he come to be reimbursed of the price paid, and the interest thence arising, reckoned at all the usual rates; the use whereof will appear in the solution of the following or like questions.

Quest. 1. A person buys an annuity for 10 years purchase: How long must he enjoy it, in order to be reimbursed of the price, reckoning interest at $3\frac{1}{2}$ *per cent.*?

Under $3\frac{1}{2}$ *per cent.* and opposite to 10 on the left, you have the answer, 12 years and 191 days.

Quest. 2. How many years possession will reimburse an annuitant, who pays $16\frac{1}{2}$ years purchase, interest at 5 *per cent.*? *Ans.* 35 years 264 days.

CHAP. XIV.

MENSURATION.

EVery magnitude is measured by some magnitude of the same kind. A line by a lineal foot, yard, &c. A superficies, or surface, by a square foot, yard, &c. A solid by a cubic foot, yard, &c.

The lineal or running measure is known to all, and needs neither direction nor example. There remains, then, the superficial and solid measure to be explained.

Superficial measure may be conceived, by imagining a floor paved with tiles, each a foot square; for then the number of tiles will be equal to the number of square feet in that floor. Now, if the floor be just one foot broad, the number of tiles, or square feet, will be equal to the number of lineal feet in the length of the floor. If the floor be two or three feet broad, the number of tiles will be twice or thrice as many. And universally, the number of feet in length multiplied by the

the number of feet in breadth will give the number of tiles or ſquare feet in the floor.

Solid meaſure may be conceived by imagining a wall built with ſtones, each a cubic foot; for then the number of ſtones will be equal to the number of cubic feet in the wall. Firſt, then, if the wall be one foot thick, and one foot high, the number of ſtones, or cubic feet, will be equal to the number of lineal feet in the length of the wall. Secondly, if the wall be of the ſame length and height, as before, viz. one foot, but two or three feet thick, then the number of ſtones, or cubic feet, will be twice or thrice as many as in the firſt ſuppoſition. Thirdly, if the length and thickneſs be the ſame as in the laſt ſuppoſition, but the height two or three feet, then the number of ſtones, or cubic feet, in the wall, will accordingly be twice or thrice as many as in the laſt ſuppoſition. And univerſally, the number of feet in length, multiplied by the number of feet in thickneſs, and that product by the number of feet in height, will give the number of ſtones, or cubic feet, in the wall.

In treating of menſuration, I ſhall enumerate the various ſorts of ſurfaces and ſolids, give ſhort deſcriptions of them, with ſuch draughts or figures as will in ſome meaſure ſhow the reaſon of the rules. In the examples, I ſhall uſe abſolute or abſtract numbers, that may repreſent any ſort of meaſure; as, inches, feet, yards, &c. After this general ſketch of menſuration, I ſhall apply the doctrine to ſurveying, to the meaſuring of artificers work, to the meaſuring of board and timber, and to gauging.

I. *Superficial Meaſure.*

Problems, ſhewing how to find the ſuperficial content, or area.

I. A ſquare, or four-ſided figure, whoſe angles are right, and ſides equal; as, A B C D. Pl. I.

Rule. Multiply the ſide into itſelf, the product is the area, or content.

Ex.

Ex. Multiply the ſide A B = 8
by itſelf 8

Area or content 64

II. A rectangle, oblong, or rectangular parallelogram, viz. a four-ſided figure, whoſe angles are right, and whoſe oppoſite ſides only are equal: as, A B C D. Pl. 1.

Rule. Multiply the length by the breadth, and the product is the area.

Ex. Multiply the length A B = 24
by the breadth B D = 8

Area 192

III. A rhombus, or four ſided figure, whoſe ſides are equal, but the angles not right; as, A B C D. Pl. 1.

Rule. Multiply one ſide into the perpendicular height, the product is the area.

Ex. Multipy the ſide A B = 9.5
by the height A E or B G = 7.8

760
665

Area 74.10

IV. A rhomboides, or parallelogram not rectangular, viz. a four-ſided figure, whoſe angles are not right, and whoſe oppoſite ſides only are equal; as, A B C D. Pl. 1.

Rule. Multiply the length by the perpendicular height or breadth, the product is the area.

Ex. Multiply the length A B = 12.6
by the height A E or B G = 4.3

50.6
$\frac{1}{3}$ = 4.2

Area 54.8

V. A

V. A right-angled triangle, or triangle wherein one of the angles is right; as A B C. Pl. 1.

Rule. Multiply the perpendicular into the baſe, half the product is the area.

Or, multiply half the perpendicular into the baſe, or half the baſe into the perpendicular, the product is the area.

Ex. Multiply the perpendicular A B = 6.4
by the baſe B C - = 8.5

$$\begin{array}{r} 320 \\ 512 \\ \hline 2)54.40 \\ \hline 27.2 \end{array}$$ area.

Or, Multiply half A B = 3.2
by the baſe B C = 8.5

$$\begin{array}{r} 160 \\ 256 \\ \hline 27.20 \end{array}$$ area.

Or, Multiply half B C = 4.25
by A B = 6.4

$$\begin{array}{r} 1700 \\ 2550 \\ \hline 27.200 \end{array}$$ area.

Note. In a right-angled triangle, if you add the ſquares of the baſe and perpendicular, the ſquare root of that ſum will be the hypotenuſe, or longeſt ſide. *Euclid* I. 47.

VI. An oblique-angled triangle, or triangle wherein none of the angles is right; as, A B C. Pl. 1.

Rule. Drop a perpendicular from any one of the angles upon the baſe, produced, if need be, as in fig. 2.; then proceed as in right-angled triangles, viz. multiply the

perpendicular into the bafe, half the product is the area.

Or, Multiply half the perpendicular into the bafe, or half the bafe into the perpendicular, the product is the area.

$$\begin{array}{lr} \textit{Ex.}\ \text{Multiply the bafe A B} = & 14.8 \\ \text{by the perpendicular C D} = & \underline{5.4} \\ & 592 \\ & \underline{740} \\ & 2)\underline{79.92} \\ & 39.96\ \text{area.} \end{array}$$

$$\begin{array}{lr} \text{Or, Multiply the bafe A B} = & 14.8 \\ \text{by half the perpendicular C D} = & \underline{2.7} \\ & 1036 \\ & \underline{296} \\ & 39.96\ \text{area.} \end{array}$$

$$\begin{array}{lr} \text{Or, Multiply half the bafe A B} = & 7.4 \\ \text{by the perpendicular C D} = & \underline{5.4} \\ & 296 \\ & \underline{370} \\ & 39.96\ \text{area.} \end{array}$$

VII. The three fides of any triangle being given, to find the area, without having recourfe to the perpendicular. Pl. 1.

Rule. Add the three given fides, and from their half-fum fubtract the three fides feverally; multiply the half-fum and the three remainders into one another continually; the fquare root of the laft product is the area.

Ex.

Ex.	A B 21			
	B C 17	24	24	24
	A C 10	10	17	21
	2)48	14 rem.	7 rem.	3 rem.
	24 half-ſum.			

And $24 \times 14 \times 7 \times 3 = 7056$. And $\dot{7}0\dot{5}6(84$ root = area.

$$\begin{array}{r} 7056(84 \\ 64 \\ \hline 164)656 \\ 656 \\ \hline \end{array}$$

VIII. A trapezium, viz. any four-ſided figure that is neither a ſquare, rectangle, rhombus, nor rhomboides; as, A B C D. Pl. 1.

Rule. Multiply the diagonal into the half-ſum of the two perpendiculars, or the contrary; the product is the area.

Ex. Multiply the diagonal B D = 28.4
by half of A a 7 + half of C a 6 = 6.5

$$\begin{array}{r} 28.4 \\ 6.5 \\ \hline 1420 \\ 1704 \\ \hline \text{Area } 184.60 \end{array}$$

IX. A parallelopleuron, or trapezium that has two of its ſides parallel, being a ſegment of a triangle cut by a line drawn parallel to the baſe; as, A B C D. Pl. 1.

Rule. Multiply the half-ſum of the two parallel ſides by the perpendicular height.

Ex. Multiply half A B 12 + half C D 22 = 17
by the perpendicular B E - = 13

$$\begin{array}{r} 17 \\ 13 \\ \hline 51 \\ 17 \\ \hline \text{Area } 221 \end{array}$$

Note. The parallelopleuron may alſo be a ſegment of a right-angled triangle; and in this caſe B D is the perpendicular, which multiplied into half the ſum of A B and C D, will give the area.

X. A polygram, or irregular polygon, viz. a figure bounded by five or more unequal ſides; as, A B C D-E F. Pl. 1.

Rule. Divide all ſuch many-ſided, multangular, and irregular figures, into trapeziums and triangles, then meaſure them by Prob. 6. and 8.

Ex. Divide the polygram A B C D E F into the trapezium A B C F, and the two triangles C D F and F D E; drop the perpendiculars A a, C a, D r, and D m; then meaſure the trapezium and triangles as taught in Prob. 8. and 6.; and the ſum of their areas will be the area of the given polygram.

XI. A regular polygon, or ordinate figure whoſe ſides and angles are all equal; as A B C D E. Pl. 1.

Such polygons take their names from the number of their ſides, or rather angles. Thus, if the polygon has 3 ſides, or angles, it is called a *trigon*, or *equilateral triangle*; if the polygon have 4 ſides, it is called a *tetragon*, or *ſquare*; if 5, a *pentagon*; if 6, a *hexagon*; if 7, a *heptagon*; if 8, an *octagon*; if 9, an *enneagon*; if 10, a *decagon*; if 11, an *endecagon*; if 12, a *dodecagon*, &c.

Rule. Biſect any two adjacent angles, and from the point where the biſecting lines meet, drop a perpendicular upon the baſe, multiply half the ſum of the ſides into the perpendicular, and the product is the area of the polygon.

Ex.

Ex. The ſide of a pentagon is	= 16.4
Multiply by $\frac{1}{2}$ the number of ſides	2.5
	820
	328
Half-ſum of the ſides	41.00
Multiply by the perpend. m n	11.3
	123
	41
	41
Area of the pentagon	463.3

Hence every polygon may be reſolved into as many equal triangles as it has ſides, the ſum of whoſe areas will be equal to the area of a right angled triangle, one of whoſe legs is the perimeter or ſum of the ſides, and the other the radius or ſemidiameter of the inſcribed circle.

If the ſide of the ſeveral regular polygons be 1, and their reſpective areas be computed by trigonometry, theſe areas may be inſerted in a table, and will be ſo many multipliers for the ready finding the area of any other like polygon. The table of areas or multipliers follows.

Names.	*Multipliers.*	*Names.*	*Multipliers*
Trigon	.433013	Octagon	4.828427
Tetragon	1.000000	Enneagon	6.181827
Pentagon	1.720475	Decagon	7.694209
Hexagon	2.598076	Endecagon	8.514250
Heptagon	3.633959	Dodecagon	9.330125

The areas of like polygons are to one another as the ſquares of their homologous or like ſides, Euclid VI. 20.; but the ſide of the polygons in the table is 1, whoſe ſquare is 1; and therefore the ſquare of the ſide of any given polygon multiplied into the tabular area of the like polygon will produce the area thereof.

Ex.

Ex. Required the area of a hexagon whoſe ſide is 10.

The ſquare of 10, or 10 × 10 = 100; and 100 × 2.598076 = 259.8076 the area ſought; and in like manner may the area of any other regular polygon be found.

XII. A circle, the moſt capacious of all figures, is bounded by one curve line, called the *circumference* or *periphery*, to which all lines drawn from the middle point or centre are equal; as A D B E. Pl. 1. fig. 1.

The line A B paſſing through the centre C, and terminated on both ſides by the circumference, is called the *diameter*; and half that line, viz. A C, drawn from the centre to the circumference, is called the *ſemidiameter*, or *radius*. The diameter divides the circle into two equal parts, called *ſemicircles*; as, A D B and A B E.

Rule. Multiply the radius into half the periphery, or the periphery into half the radius; the product is the area.

The reaſon is obvious: for a circle differs not from, or is the ſame with, a regular polygon of an infinite number of ſides.

Ex. Required the area of a circle whoſe diameter is 20, and periphery 62.8318.

Half the diameter or radius is 10; and half the periphery is 31.4159; and 31.4159 × 10 = 314.159, the area ſought.

Or, Half the radius is 5, and 62.8318 × 5 = 314.159, as before.

If the area of a ſemicircle, as A B D, be required, multiply the radius A C into half the ſemicircular arch A D B; or, multiply the ſquare of the diameter A B into .3927; either of theſe products is the area, or find the area of the whole circle, and then take its half.

If the area of a quadrant, as A C E be demanded, multiply the radius A C into half the quadrantal arch A E; or, multiply the ſquare of the diameter A B into .19635; either of theſe products is the area: or, find the area of the whole circle, and then take one fourth of it.

A

A great many other queſtions relative to the diameter, periphery, and area of circles, may occur; moſt of which may be ſolved by one or other of the proportions following.

1. As 7 to 22, nearly; or, as 113 to 355, more nearly; or, as 1 to 3.14159, ſtill more nearly; ſo the diameter of a circle to the circumference.

2. As 22 to 7; or, as 355 to 113; or, as 3.14159 to 1; or, as 1 to .31831; ſo the circumference to the diameter.

3. As 1 to 3.14159, ſo the ſquare of the radius to the area of the circle.

4. As 1 to .0795775, ſo the ſquare of the periphery to the area of the circle.

5. As 14 to 11; or, as 452 to 355; or, as 1 to .785398, or, for practice, to .7854; ſo the ſquare of the diameter, viz. the area of the circumſcribed ſquare A B C D, fig. 2. to the area of the circle.

Note. .7854 is the area of a circle whoſe diameter is unity, and circles are to one another as the ſquares of their diameters, Euclid XII. 2.

6. As 1 to .7071, ſo the diameter of a circle to the ſide of the inſcribed ſquare m n o p. Or, Multiply the ſquare of the ſemidiameter by 2, and the ſquare root of the product is the ſide of the inſcribed ſquare.

7. As 1 to .2251, ſo the periphery of a circle to the ſide of the inſcribed ſquare.

8. As 1 to 1.2732, ſo the area of a circle to the ſquare of the diameter.

9. As 1 to 12.56637; or, as .0795775 to 1; ſo the area of a circle to the ſquare of the periphery.

10. As 1 to .6366, ſo the area of a circle to the area of the inſcribed ſquare.

11. As 1 to 1.4142, ſo the ſide of a given ſquare to the diameter of the circumſcribing circle.

12. As 1 to 4.443, ſo the ſide of a ſquare to the periphery of the circumſcribing circle.

13. As 1 to .8862, ſo m n, the diameter of a circle, to

to A B, the side of a square, equal to the circle in area; fig. 3.

14. As 1 to .2821, so the periphery of a circle to the side of a square equal in area.

15. As 1 to 1.128, so the side of a square to the diameter of a circle equal to the square in area.

16. As 1 to 3.545, so the side of a square to the periphery of a circle equal in area.

XIII. A sector of a circle; as A C B E, or A C B D. Pl. 2. fig. 1.

Rule. Multiply the radius A C into half the arc A E B, the product is the area of the sector A C B E; which subtracted from the area of the whole circle, leaves the area of the sector A C B D.

Ex. Suppose the radius A C to be 10, and half the arc A E B to be 10.47198: then 10 × 10.47198 = 104.7198, the area of the sector A C B E; which subtracted from the area of the whole circle, found by Prob. 12. viz. 314.159, leaves 209.4393, the area of the sector A C B D.

If the area of the segment A B E, or A B D, be required, from the area of the sector A C B E, found above, subtract the area of the triangle A C B, and there will remain the area of the segment A B E; which subtracted from the area of the whole circle, will leave the area of the segment A B D.

The length of the arch-line of any sector or segment, as A C B, fig. 2. may be found thus.

Divide the chord line A B into four equal parts, and set one of these parts from A to D, and from B to C; join D C; which line D C will be equal to half the arch line A C B.

Or, if the arch-line be given in degrees, say, As 360, the periphery in degrees, to the periphery in measure, found by Prob. 12. so the degrees of the arch-line to the arch-line in measure.

If A B, the chord of an arc A C B, fig. 3. be given, and also the versed sine D C, the diameter of the whole circle may be found thus.

As

As D C : D A : : D A : D E.
And D E + D C = C E, the diameter.
Again, D E : D A : : D A : D C.
And the ſquare root of D C × D E = D A, or D B; Euclid III. 31. and VI. 8. Cor.

XIV. A lune, or ſpace bounded by arcs of two circles of a different radius, like the falcated moon; as, A D B E. Pl. 2.

Rule. Find C and K, the centres of the two circles A E B L and A D B M, and complete the circles.

Then, by Prob. 12. or 13. find the area of the ſemicircle or ſegment A C B E, and alſo the area of the ſegment A C B D; ſubtract the one from the other, and the remainder is the area of the lune.

XV. A ring, or ſpace included between the circumferences of two concentric circles; as, a b c d. Pl. 2. fig. 1.

Rule. Find the area of both circles, by Prob. 12.; ſubtract the leſſer from the greater; the remainder is the area of the ring.

If ſuch a portion of the ring as d be required, find the area of both ſectors, and ſubtract the leſſer from the greater.

If a portion of the ring, as b, be demanded, find the area of both ſegments, and ſubtract the leſſer from the greater.

If the area of a mixed or compound figure be required, viz. a figure bounded partly by right lines, and partly by circular arcs, as A B C D, fig. 2. reſolve ſuch a figure into triangles, trapezia, and into ſectors or ſegments; then find the area of theſe ſeverally; and their ſum will be the area of the compound figure ſought.

XVI. An ellipſe or oval, viz. any thorough plane ſection of a cone, not parallel to the baſe; as A B C D, Pl. 2. fig. 1.

Rule. Multiply the longer axis or tranſverſe diameter A B, into the ſhorter axis or conjugate diameter C D;

 then

then multiply that product by .7854; and this laſt product is the area.

Ex. Suppoſe the tranſverſe diameter A B to be 26, and the conjugate D C 20, required the area.

$26 \times 2 = 520$, and $520 \times .7854 = 408.4080$, the area of the given ellipſe.

The conic writers demonſtrate the following proportions, viz.

1. The area of any ellipſe is a mean proportional between the area of the circumſcribing and inſcribed circle, viz. a circle on the tranſverſe, and another on the conjugate diameter; as A C B D and c m d n, fig. 2.

2. As the longer axis to the ſhorter, that is, as C D (= A B) to c d, ſo any chord-line of the circle b b (parallel to c d) to r r. And as m n (= c d) to A B, ſo o o (parallel to A B) to r t.

3. As CD to c d, or as b b-to r r, ſo the area of the circular ſegment b A b to the area of the elliptical ſegment r A r.

4. As m n to A B, or as o o to r t, ſo the area of the circular ſegment o d o to the area of the elliptical ſegment r d t.

If the elliptical area included between two parallels be required, by the third or fourth of the above proportions, find the area of both ſegments, and ſubtract the leſſer from the greater.

XVII. A parabola, viz. any ſection of a cone parallel to a plane, applied to the outſide of the cone; as, A C B. Pl. 2.

Rule. Multiply the baſe, or greateſt ordinate, A B, into the perpendicular height D C, and two thirds of the product is the area, the parabola being equal to two thirds of the circumſcribing rectangle A E G B.

Ex. Suppoſe A B to be 18.5, and D C 19; required the area of the parabola.

18.5

```
    18.5
      19
   -----
    1665
    185
   -----
3) 351.5
   -----
   117.16̇
   117.16̇
   -----
   234.3̇ area.
```

Or, Multiply 351.5 by $.\dot{6} = \frac{6}{9}$.

```
     351.5
         6
    ------
 9)2109.0(
     234.3̇ area.
```

If the parabolic area included between two parallels A B and m n, be required, find the area both of the parabola A C B and m C n, and ſubtract the leſſer from the greater.

If a line be drawn from C to A, and another from C to B, the triangle A C B will be three fourths of the parabola.

XVIII. A cycloid; as A D B E. Pl. 2.

Rule. Find, by Prob. 12. the area of the circle C, deſcribed on the axis D E, multiply that area by 3, and the product is the area of the cycloid.

Note. The cycloid is formed by applying any circle, as C, to the line A B, ſo as the point n may coincide with the point A; and then, rolling the circle, as a wheel, along the line A B, till the point n complete a revolution, and coincide with B. The path deſcribed by the point n is the cycloidical curve; A B the baſe, D E the axis, the point E the vertex, and C the generating circle.

XIX. The ſurface of a cylinder, or ſolid, like a rolling ſtone, formed by the revolution of a rectangle round one of its ſides; as, A B C D. Pl. 2.

The ſurface of a cylinder, excluding the baſes, when ſpread out, becomes a rectangle, one of whoſe ſides is the periphery of the circular baſe, and the other the ſame with the height or length of the cylinder.

Rule. Multiply the periphery of the baſe into the

height,

height, and to the product add twice the area of the base; the sum is the area sought.

XX. The surface of a cone, or solid like a sugar-loaf, formed by the revolution of a right-angled triangle round one of its legs; as, A B C. Pl. 2.

The surface of a cone, excluding the circular base, when spread out, becomes a sector of a circle, whose terminating arc is the periphery of the base, and whose radius is A B or A C, the length of the outside of the cone, or hypotenuse of the generating triangle. The point A is called the *vertex*, and A o the *axis*, of the cone.

Rule. Multiply the periphery of the base into A B, the length of the side; and to half the product add the area of the base; the sum is the area sought. See Prob. 12. and 13.

If the surface of the frustum of a cone, cut by a plane parallel to the base, as m n B C, be required, find the curve surface of the whole cone A B C, and also of the cone A m n, cut off; from the former subtract the latter; to the remainder add the area of the two bases; and the sum is the area of the frustum.

The axis A o, or height of the whole cone, is found by saying, As the semidifference of the diameters of the greater and lesser base to the perpendicular height of the frustum; so the semidiameter of the greater base to A o; from which subtract the height of the frustum, and the remainder will be the height of the cone cut off; whence A m or A n may be found by the note in Prob. 5.

Or, A m may be directly found by saying, As the semidifference of the two diameters to B m, so the semidiameter of the greater base to A B; from which subtract B m, and the remainder will be A m.

The area of the outside-surface of the frustum of a cone may be found more readily thus: Add the peripheries of the two bases, and multiply half their sum by the outside-height B m, and to the product add the area of the two bases.

XXI. The

XXI. The ſurface of a ſphere or globe, viz. a round body formed by revolving a ſemicircle round its diameter; as, A B C D. Pl. 2.

Rule. Multiply the periphery of the globe into the axis or diameter A B, the product is the area.

Or, find the area of a circle whoſe diameter is the ſame with that of the globe, and multiply the area ſo found by 4.

Ex. Suppoſe the diameter A B to be 20, then, by Prob. 12. the periphery will be 62.8318; and 62.8318 × 20 = 1256.636, the area of the ſurface of the globe.

Or, The area of a circle whoſe diameter is 20, by Prob. 12. is 314.159; and 314.159 × 4 = 1256.636, as before.

If the area of the ſurface of a ſegment, as m C n, be required, ſay,

As the axis or diameter of the ſphere C D,
To the area of the whole globe;
So the height of the ſegment C r,
To the area of the ſegment, excluſive of the baſe.

If the diameter of the globe be wanted, divide the ſquare of m r, the ſemidiameter of the fruſtum's baſe by C r its height, and the quot will give r D the height of the other fruſtum, and the ſum of C r and r D is the diameter ſought. See Prob. 13.

The area of the ſegment m C n may alſo be found thus: Square m r and r C, add theſe ſquares, and the ſquare root of their ſum is C m; then find the area of a circle whoſe radius is C m, and this will be equal to the area of the given ſegment, excluſive of the circular baſe.

If the area of that part of the ſurface of a ſphere that lies between two parallel planes, as m n, and a o, be demanded, it may be found thus: Find, as above directed, the area of the two ſegments m C n and a C o, ſubtract the leſſer area from the greater, and the remainder is the area ſought.

XXII. A ſpherical triangle, or triangle formed on the ſurface of a globe by three great circles, viz. circles whoſe

whose centre is the same with that of the globe; as, A B C. Pl. 2.

Rule. From the sum of the three angles subtract 180 degrees, multiply the area of the whole surface of the globe by the remainder, divide the product by 720, the quot is the area of the triangle.

E X A M P L E.

		°	′
Suppose the sum of the angles $A + B + C =$		225	10
	Subtract	180	
	Rem.	45	10

Suppose again the diameter of the given sphere or globe to be 20, then the area of the whole surface, as found above, will be 1256.636.

Multiply	1256.636
by the rem.	45.1$\cancel{6}$
	1256636
	6283180
	5026544
	56674.2836
	418.8786$\cancel{6}$
	418.8786$\cancel{6}$
	720)57512.0409$\cancel{3}$(79.877 area of the triangle.

The surfaces of prisms, pyramids, and other regular bodies, consist of triangles, parallelograms, or polygons; and so the way of measuring them is taught in the preceding problems.

II. *Mensuration of Solids.*

Problems, shewing how to find the solid content, or solidity.

XXIII. A cube, or solid contained under six equal squares; as, A B C D. Pl. 3.

Rule.

Rule. Multiply the ſide of the cube into itſelf, and that product again by the ſide; this laſt product is the ſolid content or ſolidity of the cube.

Ex. Multiply the ſide A D	=	4
by itſelf -	=	4
Again, Multiply this product		16
	by	4
Solidity of the cube		64

XXIV. A priſm, or ſolid whoſe baſes are equal ſimilar parallel triangles, ſquares, parallelograms, or polygons, and the ſides all parallelograms.

A priſm whoſe baſes are ſquares or parallelograms, is uſually called a *parallelopiped.*

When the baſes are triangles, the ſolid is called a *triangular priſm,* and if the baſes are polygons, it is called a *polygonal priſm.*

Rule. Find the area of the baſe as taught in ſuperficial meaſure, and multiply that by the length of the priſm or parallelopiped.

Ex. Required the ſolid content of the triangular priſm A B C D. Pl. 3.

Multiply B C	=	18
by half the per. m n	=	6
Area of the baſe		108
Multiply by the length A B	=	40
Solidity of the priſm		4320

XXV. A cylinder. See a deſcription of this ſolid, Prob. 19. Pl. 3. fig. 1.

Rule. The ſolidity of a cylinder is found the ſame way as that of a priſm or parallelopiped, viz. Find the area of the circular baſe by Prob. 12. multiply that by the length or height, the product is the ſolid content.

Ex. Required the ſolidity of a cylinder, the diameter of whoſe baſe is 9, and length 15.

$9 \times 9 \times .7854 \times 15 = 954.261$ the ſolidity.

If it be required to find how much liquor is contained in a cylindrical veſſel that is in part empty, lying with its axis parallel to the horizon; ſuch as a veſſel whoſe baſe is the circle A B C D, fig. 2. the empty part being the ſegment A B D; find by Prob. 13. the area of the ſegment B C D, and multiply this area by the length.

XXVI. A pyramid, or ſolid whoſe baſe is a triangle, ſquare, parallelogram, or polygon, and its ſides all plane triangles, terminating in a point; as, A D B. Pl. 3.

Rule. Find the area of the baſe, and multiply that into one third of the altitude or height; the product is the ſolid content, a pyramid being equal to one third of the circumſcribing priſm. Euclid XII. 7.

Ex. Suppoſe the area of the baſe A B to be 4.25
Multiply that by $\frac{1}{3}$ of D C = 5.5

$$\begin{array}{r} 4.25 \\ 5.5 \\ \hline 2125 \\ 2125 \\ \hline 23.375 \end{array}$$

Solidity of the pyramid 23.375

If the pyramid be cut by a plane m n parallel to the baſe A B, and the ſolidity of the fruſtum A m n B be required; from the ſolidity of the whole pyramid A-D B ſubtract the ſolidity of the pyramid m D n cut off, the remainder is the ſolidity of the fruſtum.

To find the height of the whole pyramid, ſay, As the difference between one of the ſides of the greater baſe and the correſpondent ſide of the leſſer baſe, to the height of the fruſtum, ſo the ſide of the greater baſe to the height of the whole pyramid; from which ſubtract the height of the fruſtum, and the remainder will be the height of the pyramid cut off.

The ſolidity of the fruſtum of a pyramid may alſo be found by one or other of the three rules following.

Rule 1. To the product of any two correſpondent ſides

ſides of the two baſes, add the ſquares of theſe ſides; multiply that ſum by one third of the fruſtum's height, and the product will be the ſolidity, if the baſes are ſquares.

Rule 2. Multiply the areas of the two baſes together, and to the ſquare root of the product add theſe two areas; multiply that ſum by one third of the fruſtum's height, and the product is the ſolidity of the fruſtum, whether the baſes be triangles, ſquares, or polygons.

Rule 3. To the product of any two correſpondent ſides of the two baſes, add one third part of the ſquare of their difference; and that ſum will be the area of a mean baſe; which multiplied into the height, gives the ſolidity of the fruſtum, whether the baſes be triangular, ſquare, or multangular.

A ſolid, reſembling the fruſtum of a pyramid, and having parallel baſes, and theſe baſes both rectangles, or the one a rectangle and the other a ſquare, but diſproportional, the ſides of the one baſe not having the ſame proportion to one another as the correſpondent ſides of the other, is called a *priſmoid*; as, ABC. Pl. 3. fig. 2. And its ſolidity may be found as follows.

To the longeſt ſide of the greater baſe add half the longeſt ſide of the leſſer baſe, and multiply the ſum by the breadth of the greater baſe, and reſerve the product.

Again, to the longeſt ſide of the leſſer baſe add half the longeſt ſide of the greater baſe; and multiply the ſum by the breadth of the leſſer baſe; and to this product add the product formerly reſerved; and multiply the ſum by a third part of the height, and the product is the ſolidity of the priſmoid.

If the area of the outſide ſurface of the priſmoid be required, multiply half the ſum of the perimeters of both baſes by the outſide height, and to the product add the area of the two baſes.

XXVII. A cone. See this ſolid deſcribed, Prob. 20.

A cone is one third of the circumſcribing cylinder; or, A cone is the ſame with a pyramid of an infinite number of ſides: and the ſolidity is found the ſame way, viz.

Multiply the area of the base by one third of the altitude or perpendicular height; Euclid XII. 10.

If the cone be cut by a plane m n parallel to the base, the solidity of the frustum m n B C is also found by subtracting the solidity of the cone cut off A m n from the solidity of the whole cone A B C. Pl. 3. See Prob. 20.

The solidity of the frustum of a cone may also be found by any of the three rules following.

Rule 1. To the product of the diameters of the two bases add the squares of the said two diameters, and multiply the sum by .7854, the product will be the triple area of a mean base; which multiplied by one third of the perpendicular height, will give the solidity of the frustum.

Rule 2. Multiply the area of the greater base into the area of the lesser; extract the square root of the product; to this root add the areas of the two bases; and multiply the sum by one third of the frustum's height, and the product is the solidity.

Rule 3. To the product of the greater and lesser diameters add one third part of the square of their difference; and multiply the sum by .7854; the product is the area of a mean base; which multiplied by the perpendicular height, the product is the solidity.

If the base of a cone, or of a cylinder, be an ellipse at right angles to the axis, find the area of the base by Prob. 16.; and then proceed to find the solidity, as taught above. The solidity of the frustum of such a cone may also be found as above; only find the area of the elliptical bases by Prob. 16.

A solid, resembling the frustum of a cone, and having parallel bases, and these bases both elliptical, or the one base an ellipse, and the other a circle, but disproportional, the diameters of the one base not having the same proportion to one another as the correspondent diameters of the other base, is called a *cylindroid*; as, A B C. Pl. 3. fig. 2. And its solidity may be found thus.

To the longest diameter of the greater base add half the longest diameter of the lesser base, and multiply the sum

sum by the shortest diameter of the greater base, and reserve the product.

Again, to the longest diameter of the lesser base add half the longest diameter of the greater base, and multiply the sum by the shortest diameter of the lesser base; and to this product add the product formerly reserved, and that sum will be the triple square of a mean diameter; which multiply by .7854; and again multiply that product by a third part of the height; and this last product is the solidity of the cylindroid.

If the area of the outside surface of the cylindroid be required, measure the periphery of both bases, and multiply half their sum by the outside height, and to the product add the area of the two bases.

XXVIII. A sphere or globe. See the description of this solid, Prob. 21. Pl. 3.

Rule. A sphere or globe is equal to two thirds of the circumscribed cylinder; that is, a cylinder of equal diameter and altitude with the globe; and therefore find the area of a great circle; multiply this by the diameter; two thirds of the product is the solid content.

Ex. Suppose the diameter of a sphere or globe to be 20, then, by Prob. 12.

The area of the circle will be -	314.159
Multiply by the diameter, -	20
	3)6283.180
	2094.393
	2094.393
Solidity of the globe, -	4188.786

A sphere may be considered as composed of an infinite number of cones, having their bases in the surface, and their common vertex in the centre; and so the solidity of a sphere may also be found thus: Multiply the diameter into the circumference; the product is the superficial content or area: then multiply this area by one

 sixth

ſixth of the diameter, or by one third of the radius; the product is the ſolid content.

Ex. Suppoſe the diameter of a globe to be 20; then, by Prob. 12.

The periphery will be - -	62.8318
Multiply by the diameter, -	20
	3)1256.636
Multiply by $\frac{1}{6}$ diameter, =	3.3
	3769.908
	418.8786
Solidity, as before, - -	4188.7866

The ſolidity of a ſphere may be found nearly, by multiplying the cube of the diameter into .5236, or, for greater accuracy, into .523598, the ſolidity of a ſphere whoſe diameter is unity.

Thus, the cube of the diameter 20 is $20 \times 20 \times 20 = 8000$; and $.5236 \times 8000 = 4188.8$ the ſolidity nearly; or, $523598 \times 8000 = 4188.784$, more nearly.

XXIX. The polar ſegment, or fruſtum of a ſphere; as, A C B D. Pl. 3.

Rule. From the triple product of the diameter of the ſphere C E into the ſquare of the fruſtum's height C D, ſubtract twice the cube of the ſegment's height C D; then divide the remainder by 1.91; or, multiply by .52356, or, for practice, by .5236, the quot or product is the ſolid content.

Ex. Suppoſe C E = 20; and C D = 5.

Then $5 \times 5 \times 20 \times 3 - 5 \times 5 \times 5 \times 2 = 1250$.

And 1.91)1250(654.45, the ſolidity ſought.

Or, $1250 \times 52356 = 654.45$, as before.

The ſegment A C B D ſubtracted from the whole ſphere, leaves the greater ſegment A E B D.

XXX. The

XXX. The middle ſegment or zone of a ſphere; as, A B C D. Pl. 3.

Rule. Multiply the ſquare of the ſphere's diameter E G by 2, and to the product add the ſquare of the diameter of the baſe A B or C D; divide this ſum by 3.82; or, for greater accuracy, by 3.8197; and multiply the quot by the zone's thickneſs, or height m n, the product is the ſolid content.

Ex. Suppoſe E G = 20, A B or C D = 17.32, and m n = 10.

Then $20 \times 20 \times 2 + 17.32 \times 17.32 = 1099.9824$

And 3.82)1099.9824(287.95

And $287.95 \times 10 = 2879.5$ the ſolidity of the zone.

The ſolidity of the zone A B C D may alſo be found by ſubtracting twice the ſolidity of the polar ſegment A E B m from the ſolidity of the whole ſphere.

Hence may be found the ſolidity of any ſegment of a ſphere included between two parallel planes; as, a o B A; viz. Find the ſolidity of the ſegment A E B, and alſo that of the ſegment a E o, and ſubtract the leſſer from the greater.

XXXI. A ſector of a ſphere; as, A B D C. Pl. 3.

Rule. A ſpherical ſector, as already obſerved, conſiſts of an infinite number of cones, having their baſes in the ſurface of the ſphere B D C, and their common vertex in the centre A: Wherefore, by Prob. 21. find the area of the ſegment B D C; and multiply this area into the third part of A B the radius of the ſphere.

Hence we have another method of finding the ſolidity of a ſpherical ſegment, ſuch as B D C, viz. From the ſector A B D C ſubtract the cone A B C, and there will remain the ſolidity of the ſegment. But if the ſpherical ſegment be greater than a hemiſphere, the cone muſt be added to the ſector.

XXXII. A ſpheroid, or ſolid generated by revolving a ſemiellipſe round its axis; as, A B C D. Pl. 3.

If

If the ſemiellipſe be revolved round the longer axis A B, the ſolid thence generated is called an *oblong ſpheroid* and reſembles an egg.

If the revolution be round the ſhorter axis C D, the ſolid thence ariſing is called an *oblate ſpheroid,* being ſomewhat like a bias-bowl.

Rule. Multiply the axis round which the ſemiellipſe was revolved into the ſquare of the other axis; then multiply this product by .5236; this laſt product is the ſolidity of the ſpheroid, whether oblong or oblate.

Example. Suppoſe C D = 5.5; and A B = 9.

Then 5.5 × 5.5 × 9 × .5236 = 142.5501 the ſolidity of the ſpheroid.

A ſpheroid is two thirds of a cylinder whoſe height is the axis round which the ſemiellipſe was revolved, and whoſe diameter is the other axis; and accordingly the ſolidity of the above ſpheroid may alſo be found thus:

5.5 × 5.5 × .7854 × 9 = 213.82515; whereof two thirds is 142.5501, as before.

If the ſolidity of a ſegment, as, A m n, or of the middle fruſtum m n o p, be required, imagine a ſphere deſcribed on the axis A B, and cut into ſegments ſimilar to thoſe of the ſpheroid, by extending the planes m n and o p; and then ſay, As the ſquare of the axis A B to the ſquare of the axis C D, ſo the ſolidity of the ſpherical ſegment or fruſtum found by Prob. 29. or 30. to the ſolidity of the ſegment or fruſtum of the ſpheroid. *N. B.* Twice the ſegment A m n ſubtracted from the whole ſpheroid, leaves the middle fruſtum or zone m n o p.

Note. The middle fruſtum of the ſpheroid m n o p differs little from the like fruſtum of a ſphere deſcribed on the axis C D, and may have its ſolidity computed in the ſame manner, viz. To twice the ſquare of C D add the ſquare of m n or o p, divide the ſum by 3.82, and multiply the quot by a r, the height.

XXXIII. A parabolic conoid, or solid, found by revolving a semiparabola A n C round its axis A o; as, A B C. Pl. 3.

Rule. A parabolic conoid is one half of the circumscribed cylinder; multiply therefore the area of the circular base B C into one half of the altitude A o, the product is the solid content.

Ex. Suppose the diameter of the base B o C to be 12, and the height A o to be 18: Required the solid content.

$12 \times 12 \times .7854 \times 9 = 1017.8784$ solidity sought.

If the parabolic conoid be cut by a plane m n parallel to the base B C, the solidity of the frustum m n B C may be found thus: To the square of the diameter of the greater base B C add the square of the diameter of the lesser base m n; divide the sum by 2.5464; or, multiply by .3927; and then multiply the result by the height of the frustum; the product is the solid content. The frustum A m n cut off is a complete conoid, and its solidity is found as taught above.

XXXIV. A parabolic spindle, or solid generated by revolving a parabola A B C round its ordinate or base A C; as, A B C D. Pl. 3.

Rule. The parabolic spindle is equal to eight fifteenths of the circumscribing cylinder; find therefore the solidity of a cylinder, the diameter of whose base is B D, the greatest diameter of the spindle, and its height AC, the length of the spindle, or ordinate of the parabola, and $\frac{8}{15}$ of this is the solid content of the spindle.

Example. Suppose B D = 6.5, and A C = 8.

Then $6.5 \times 6.5 \times .7854 \times 8 \times \frac{8}{15} = 141.58144$ solidity.

Or, Multiply the square of B D by A C, and that product by $.41888 = \frac{8}{15}$ of .7854.

Thus, $6.5 \times 6.5 \times 8 \times .41888 = 141.58144$, as before.

The

The ſolidity of the middle fruſtum of the ſpindle, viz. m n o p is found thus: To twice the ſquare of the greateſt diameter B D add the ſquare of the diameter of the baſe, viz. m n or o p, and from the ſum ſubtract four tenths of the ſquare of the difference between theſe two diameters; divide the remainder by 3.82; or, for greater accuracy, by 3.8197; and multiply the quot by t r, the height of the fruſtum, the product is the ſolidity.

Or thus, Multiply the ſquare of the greateſt diameter by 1.5708; and multiply the ſquare of the leſſer diameter by .7854; and multiply the ſquare of the difference of theſe diameters by .31416; from the ſum of the two former products ſubtract the latter product; and multiply the remainder by one third part of the height; and that product will be the ſolidity of the middle fruſtum required.

If the ſolidity of the middle fruſtum be ſubtracted from that of the whole ſpindle, half the remainder will be the ſolidity of the fruſtum A m n or C o p.

XXXV. The five regular or Platonic bodies, Pl. 4. viz.

1. A tetraedron, or ſolid contained under four equal equilateral triangles.

This ſolid is a pyramid on a triangular baſe, and its ſolidity may be found by Prob. 26.

2. An hexaedron, or cube, viz. a ſolid contained under ſix equal ſquares.

This ſolid is a priſm; and its ſolidity may be found by Prob. 23. or 24.

3. An octaedron, or ſolid contained under eight equal equilateral triangles.

This ſolid conſiſts of two pyramids on the ſame ſquare baſe, and of the ſame height; and conſequently its ſolidity may be found by Prob. 26.

4. A dodecaedron, or ſolid contained under twelve equal equilateral pentagons.

This ſolid conſiſts of twelve equal pyramids, having pentagonal baſes, whoſe common vertex is in the middle

dle point or centre; and ſo its ſolidity may be found by Prob. 26.

5. An icoſaedron, or ſolid contained under twenty equal equilateral triangles.

This ſolid conſiſts of twenty equal pyramids having triangular baſes, and their common vertex in the centre; and ſo its ſolidity may likewiſe be found by Prob. 26.

But the area, or ſuperficial content, as well as the ſolidity of any of theſe bodies, may eaſily and readily be found from the following table.

Names.	*Area.*	*Solidity.*
Tetraedron	1.732051	0.1178511
Hexaedron	6.000000	1.0000000
Octaedron	3.464102	0.4714045
Dodecaedron	20.645729	7.663119
Icoſaedron	8.660254	2.181695

The table exhibits the area and the ſolidity of any of the above bodies, the ſide being unity. In uſing the table obſerve the following rules.

1. Multiply the proper tabular number by the ſquare of the given ſide; the product is the area, or ſuperficial content.

2. Multiply the proper tabular number by the cube of the given ſide; the product is the ſolidity.

Ex. Required the area and ſolidity of a dodecaedron, the ſide being 3.

$$3 \times 3 \times 20.645729 = 185.811561 \text{ the area.}$$

$$3 \times 3 \times 3 \times 7.663119 = 206.904213 \text{ the ſolidity.}$$

XXXVI. Any irregular body, ſuch as, the bones in a horſe's head, a thorn or whin buſh, &c.

Rule. Take any veſſel of a regular form, ſuch as that of a priſm or cylinder, and therein put the irregular ſolid; then pour in as much water as will cover the ſolid; this being done, take out the ſolid, and obſerve how far

 the

the water ſinks or falls on the ſide of the veſſel, and compute the ſolidity by Prob. 24. or 25.

Or, Take any ſort of veſſel, fill it with water to the brim; then immerſe the irregular body; receive the water that runs over, and pour it into ſome veſſel of a regular form; and then proceed as above.

III. *Surveying.*

A ſurveyor, or perſon who meaſures land, ought to be provided with a theodolite and plain table, a caſe of inſtruments, with a ſet of plotting ſcales, and a gunter's chain.

The gunter's chain is divided into 100 equal links, and is in length equal to 4 rods or poles; and 160 ſquare rods, or 10 ſquare chains, make an acre.

The Engliſh rod or pole is $16\frac{1}{2}$ feet; and ſo the length of the Engliſh chain is 66 feet, or 22 yards. The Scotch rod, pole, or fall, is $18\frac{1}{2}$ feet, and the length of the chain 74 feet. And 4 Scotch acres are nearly equal to 5 Engliſh acres.

My deſign in this place is not to teach ſurveying at any great length, but only to give a general notion of the art, ſo far as to enable the reader to meaſure any park, or plain field, and that with accuracy. For this purpoſe it will be neceſſary to ſhow what lines are to be meaſured, how the meaſures ought to be taken, and in what manner the number of acres is computed.

If the ground to be meaſured lie in the form of a rectangle, you have only to multiply the length into the breadth; only obſerve, that if the two ends differ ſomewhat in breadth, you may add them, and take half their ſum for a mean breadth; and do the ſame with reſpect to the ſides, if they happen not to be equal.

But moſt fields lie in the form of ſome irregular polygon; and in this caſe you muſt go round the field, and erect a pole at every angle. Then make out an eye-draught, viz. a figure on paper as near the ſhape of the field as can be done by the gueſs of the eye, and divide this

this draught into triangles, trapezia, or other figures, as the ſhape of the draught and field directs. Draw alſo, in the triangles and trapezia, lines to repreſent the perpendiculars. Now, the lines to be meaſured are the diagonals and perpendiculars. But before they be meaſured, the points in the field where they interſect one another muſt be found as follows.

Provide yourſelf with a theodolite, or, in lieu thereof, with a croſs, repreſented in Pl. 4. fig. 1. viz. a ſquare board, with four pins or pegs erected in the corners, ſo that A B may be perpendicular, or at right angles, to C D; and place the croſs horizontally on the top of a ſtaff F O faſtened into a hole in the middle of the board at O. By means of this croſs, find the points in the field where the perpendiculars cut the diagonals, in the manner following.

Suppoſing the polygram A B C D E F G, in Pl. 4. fig. 2. to repreſent the field, you muſt ſtep in a ſtraight line from G to B; and when you come to the place where you imagine the perpendicular A m will cut the diagonal G B, ſet up the croſs directing C D to B; and, if now B A on the croſs point directly to A in the field, you have gueſſed right; but if it do not, go further on toward B, or return back toward G, till you hit upon the place, and there fix a pole. In like manner, find the points where all the other perpendiculars cut the diagonals, and mark the places thus found by fixing poles in them.

Then meaſure the ſeveral ſegments into which the diagonals are divided, and alſo the perpendiculars; and ſet down the meaſures ſo taken in a field-book, or on the eye-draught deſcribed above. When you come home, delineate the whole on paper, from a ſcale of equal parts, draw the outlines A B, B C, &c. and caſt up the content, as follows.

The area of a triangle is found by Prob. 6. and that of a trapezium by Prob. 8. as follows.

For the triangle A B G,

Multiply the base G B = 11 + 17.20	=	28.2
by half the perpendicular A m	=	9
	Area	253.8

For the trapezium B C F G,

Multiply the sum of the perpendiculars B n + F r	=	34.54
by half the diagonal G C	=	25
G C = 19 + 5 + 26 = 50		17270
		6908
	Area	863.50

For the trapezium F C D E,

Multiply the sum of the perpendiculars D o + F p	=	31.86
by half the diagonal E C	=	23.02
E C = 13.5 + 14 + 18.54 = 46.04		6372
		9558
		6372
	Area	733.4172
To which add	B C F G	863.5
	A B G	253.8
Area of the field,	-	1850.7172

The area of the field thus computed is 1850 square chains, and 7172 square links; and, because 10 square chains make an acre, the square chains are reduced to acres by dividing by 10; that is, by moving the decimal point one place toward the left; and the figures on the right are decimal parts of an acre, as follows.

The

The decimal is reduced to value by multiplying by 4, by 40, by $30\frac{1}{4}$, and by 9, as the table of land-measure directs.

Acres	185.07172
	4
Roods	.28688
	40
Poles or Rods	11.47520
	$30\frac{1}{4}$
	14.256
	.1188
Square yards	14.3748
	9
Square feet	3.3732

Some set down the measures taken in the field, without inserting any decimal point between the chains and the links; and in this way the area of the above field will be 18507172 square links: which are easily reduced to acres; for the chain consists of 100 links, and consequently the square chain will contain 10000 square links, and 10 square chains make an acre; therefore the acre will contain 100000 square links. Divide then the area given in square links by 100000, and the quot will be acres; that is, cut off by a decimal point five places on the right, the figures on the left are so many acres, and the figures on the right are decimal parts of an acre. Thus, 18507172 square links becomes 185.07172 acres, as above.

It is usual to measure gardens, or small inclosures, with a rod-pole, viz. a rod one pole long; and in this case the rod-pole should be divided into 100 equal parts, that the parts in any measured line above a just number of rods may be decimal parts of a rod. Some use a half-rod-pole divided into 50 equal parts.

If we suppose the above field to have been measured by a rod-pole, the area will be 1850.7172 square rods, which are reduced to acres by dividing by 160, the number of square rods in an acre; or you may divide the square rods by 40, and the quot will be roods; and then divide

divide the roods by 4, and this last quot will be acres, as under.

Acres.	Or thus :
16|0)1850.7172(11.5669825	4|0)1850.7172(
4	4)46.26793
Roods 2.2679300	Acres 11.5669825
40	
Poles 10.7172	
$30\frac{1}{4}$	
21.516	
.1793	
Sq. yards 21.6953	
9	
Sq. feet 6.2577	

If a piece of ground lie in the form of a ridge, take the breadth at several places, add these several breadths, and divide their sum by the number of breadths taken, the quot will be a mean breadth; which multiplied into the length, will give the area.

If the measures of a field be taken in feet, the area will be square feet; which you may reduce into English acres by dividing by 43560, the number of square feet in an English acre; or it may be reduced to Scotch acres by dividing by 54760, the number of square feet in a Scotch acre.

If the measures of a field be taken in yards, the area will be square yards; and these may be reduced to English acres by dividing by 4840, the number of square yards in an English acre.

If the measures of an island, kingdom, or empire, be taken in English miles, the area will be square miles; and these may be reduced to English acres by multiplying by 640, the number of acres in a square mile.

And if the number of English acres on the whole surface of the terraqueous globe be required, find by Prob. 21.

21. the area of the furface of a globe, whofe diameter is 8000 Englifh miles, and multiply the area in fquare miles fo found by 640.

IV. *Artificers Work.*

1. *Carpenters Work.*

Carpenters meafure their work by the fquare of 100 feet, viz. a fquare whofe fide is 10 feet. Their meafurable work confifts chiefly of flooring, partitioning, and roofing. See Prob. 2.

Ex. 1. If a floor be 38 feet 6 inches long, and 17 feet 9 inches broad, how many fquares are in that floor?

Decimally.	*Or thus:*		
	F.	*in.*	
17.75	38	6	
38.5	17	9	
8875	654	6	
14200	28	10	6
5325			
100)6.83375	100)6|83	4	6

Anf. 6 fquares, 83 feet.

Ex. 2. How many planks or deals that are 9 feet long, and 8 inches broad, will floor a houfe that is 47 feet 6 inches long, and 24 feet 4 inches wide?

Inches	
8 = .6	24.3
Length 9	47.5
Product 6.0	1216
	17033
	97333
	6)1155.83(192.638
	9
Anf. 192 planks, $5\frac{3}{4}$ feet.	5.750

Or,

Or, Reduce the dimenſions to inches, then multiply the length of the houſe into the breadth, and divide the product by the ſquare inches in one plank.

Ex. 3. How many ſquares are contained in a partition that is 64 feet 6 inches long, and 12 feet 3 inches high?

12.25	*F.*	*in.*	
64.5	64	6	
6125	12	3	
4900	774		
7350	16	1	6
100)7.90125	100)7|90	1	6

Anſ. 7 ſquares, 90 feet.

Ex. 4. How many ſquares of roofing will cover a houſe, whoſe length within the walls is 48 feet 6 inches, and breadth 18 feet 3 inches?

It is a received rule amongſt workmen, That the flat and half-flat of any houſe, taken within the walls, is equal to the roof, though the meaſure of the roof varies a little, as the roof falls below or riſes above the true pitch; that is, as the angle at which the two ſides of the roof meets, is more or leſs than a right angle.

18.25	*F.*	*in.*	
48.5	48	6	
9125	18	3	
14600	873	0	
7300	12	1	6
Flat 885.125	Flat 885	1	6
Half 442	Half 442		
100)13.27	100)13|27		

Anſ. 13 ſquares, 27 feet.

2. *Bricklayers Work.*

Bricklayers work conſiſts chiefly of tiling; which is meaſured

meaſured by the ſquare of 100 feet; and walling or chimney work, which are meaſured by the ſquare rod of 272.25 ſquare feet, the ſide of the ſquare being 16.5 feet. But, in practice, the integer 272 is generally eſteemed ſufficiently accurate.

In roofs covered with tile, it is uſual to reckon the eaves double, which is done by adding the depth of the eaves to the whole depth of the roof.

Ex. 1. There is a roof covered with tiles, whoſe depth on both ſides, the eaves being reckoned double, is 35 feet 3 inches, and the length 48 feet: How many ſquares of tiling are in it?

		F.	*in.*
35.25		35	3
48		48	0
28200		12	
14100		280	
100)16.9200		140	
		100)16|92	

Anſ. 16 ſquares, 92 feet.

Ex. 2. A brick-wall is 598 feet long; 9 feet high, and $1\frac{1}{2}$ brick in thickneſs; How many roods of work are in it?

598
9
272.25)5382.00(19 rods.
27225
265950
245025
68.06) 209.25(3 quarters of a rod.
20418
5.07

Anſ. 19 rods 3 quarters 5 feet.

Brick and half is reckoned ſtandard thickneſs; and if the thickneſs of a wall be any other than brick and half, it may be reduced to ſtandard thickneſs by the following

R U L E.

Multiply the number of ſuperficial feet contained in the wall by the number of half-bricks in the thickneſs, and one third of the product will be the content reduced to ſtandard thickneſs.

Ex. 3. A brick-wall is 84 feet 6 inches long, and 17 feet 3 inches high, and 5 bricks and a half thick: How many rods of brick-work will be therein, when reduced to ſtandard-thickneſs?

```
          17.25
           84.5
        -------
           8025
          6900
        13800
        -------
        1457.625
              11
        --------
         1457625
        1457625
       ---------
      3)16033.875
       ----------
272.25)5344.625(19 rods.
        27225
        -----
        262212
        245025
        ------
     68)171.875(2 quarters.
        136
        ------
         35.875
```

Anſ. 19 rods 2 quarters 35 feet.

But the operation may be ſhortened, by conſtructing a table of diviſors, ſuited to the number of half-bricks in the thickneſs, as follows.

Divide 3, the number of half-bricks in $1\frac{1}{2}$, by the number of half-bricks in the thickneſs, and the quot will be a diviſor for giving the anſwer in feet. And, to

to obtain a divifor that will give the anfwer in rods, multiply 272.25 by the divifors found for feet. The table follows.

Thicknefs of the wall in bricks and half-bricks.	*Divifors giving the anfwer in feet.*	*Divifors giving the anfwer in rods.*
1	1.5	408.375
1½	1	272.25
2	.75	204.1875
2½	.6	163.35
3	.5	136.125
3½	.4285	116.659
4	.375	102.0937
4½	.3	90.75
5	.3	81.675
5½	.27	74.25
6	.25	68.09375

The former example done by the table follows.

```
74.25)1457.625(- - - - - 19.63  rods.
       7425                 4
       ----              ----
       71512             2.52  quarters.
       66825               68
       -----             ----
        46875             416
        44550             312
        -----            ----
         23250           35.36 feet.
         22275
         -----
           975
```

If a brick chimney ftand alone by itfelf, it muft be girt round; and this is to be efteemed the length; the height of the ftory is the breadth; and the thicknefs is the fame with that of the jambs. Every ftory is meafured by itfelf, and the dimenfions taken in the fame man-

 ner;

ner; and the ſhaft or ſtalk above the roof is alſo meaſured by itſelf, and the content found by multiplying the girt into the height.

Ex. 4. The girt of a brick chimney in a low ſtory is 32 feet 9 inches, and the height of the ſtory 12 feet 6 inches: How many ſquare feet of work are in the ſame?

				F.	in.	
	32.75			32	9	
	12.5			12	6	
	16375			393		
	6550			16	4	6
	3275					
Feet	409.375		Feet	409	4	6

If it be required to reduce the ſquare feet thus found to feet or rods of ſtandard thickneſs, divide by the diviſor that ſuits the thickneſs of the jambs. And if the chimney be eſteemed double work, as moſt chimneys are, double the reſult.

If a chimney have two or more funnels, the fetters or bridges that ſeparate the funnels muſt be meaſured, and their content found by multiplying the length into the breadth.

If a chimney do not ſtand by itſelf, but be placed in a gavel or ſide-wall, the back of the chimney in this caſe is not meaſured, but accounted part of the gavel or ſide-wall; and the length of the chimney is found by adding the depth of the two jambs to the horizontal extent of the breaſt.

3. *Plaiſter-work.*

Plaiſter-work is either on the roof, called *ceiling*, or on the partitions or walls of a building; and is all meaſured by the ſquare yard, containing 9 ſquare feet.

Ex. 1. If a ceiling be 54 feet 9 inches long, and 22 feet 6 inches broad, how many yards are in it?

		F.	in.	
54.75		54	9	
22.5		22	6	
27375		16	6	
10950		108		
10950		108		
9)1231.875		27	4	6
Yards 136.875		1231	10	6

Ex. 2. The length of a partition plaiftered on both fides is 44 feet 6 inches, and the height of the room 9 feet 3 inches: How many yards of plaifter-work are in it?

		F.	in.	
44.5		44	6	
9.25		9	3	
2225		4	6	
890		396		
4005		11	1	6
9)411.625		411	7	6
Yards 45.736				

4. *Joiners Work.*

Joiners meafure their work alfo by the fquare-yard; but in taking the height of any room, they ufe a fmall cord, with which they gird over the mouldings, panels, &c. alledging they ought to meafure where their plane touches; but in meafuring round the room, they take it as it is upon the floor.

Doors window-fhutters, and fuch work as is wrought on both fides, are generally efteemed work and half-work.

Example. If the wainfcotting of a room, girt downward over the mouldings, be 12 feet 9 inches high, and 118 feet 3 inches in compafs, how many fquare yards of work are in it?

118.25

		F.	in.	
118.25		118	3	
12.75		12	9	
59125		1419		
82775		59	1	6
23650		29	6	9
11825				
9)1507.6875		1507	8	3
Yards 167.52083				

The doors and windows of the above room must again be measured by themselves, and the half of their content must be added to the content of the whole room found above.

The content of chimneys, and other void places, must also be measured by themselves, and their content is to be deduced.

5. *Painters Work.*

Painters work is also measured by the square-yard, and the height is likewise taken by girding over the mouldings and panels with a small cord.

Ex. 1. A room is painted, whose height, taken by girding over the mouldings, is 14 feet 6 inches, and the compass of the room 84 feet 6 inches: How many yards of painted work are in it?

		F.	in.
84.5		84	6
14.5		14	6
4225		343	
3380		84	
845		42	3
9)1225.25		1225	3
Yards 136.138			

Ex. 2. A round pillar is painted, whose height is 18 feet 4 inches, and the girt round 10 feet 6 inches: How many yards of work are in it?

18.3

		F.	in.
18.3		18	4
10.5		10	6
916		18	4
18333		9	2
9) 92.50		192	6
Yards 21.38			

6. *Glasiers Work.*

Glasiers measure their work by the foot square, and take their dimensions to a quarter of an inch, the inch being supposed to be divided into 12 equal parts.

But it is more usual to measure by a foot decimally divided, and then the length or breadth consists of so many feet and decimal parts of a foot.

Round or oval windows, half-rounds, &c. are always measured at their full height and breadth, in consideration of the waste of glass, and trouble such work is attended with.

Ex. 1. Suppose 6 panes of glass, each 4 feet 7 inches 3 quarters long, and 1 foot 5 inches 1 quarter broad, how many feet of glass are in the said 6 panes?

		F.	in.	p.		
	4.646	4	7	9		
	1.437			1	5	3
	32522		1	1	11	3
	13938	1	11	2	9	
	18584	4	7	9		
	4646					
	6.676302	6	8	1	8	3
	6					6
Feet	40.057812	40	0	10	1	6

Ex. 2. Suppose 20 panes of glass, which measured by a decimal foot are each 2.458 feet long, and 1.75 feet broad, how many feet of glass do they contain?

2.458

```
      2.458
       1.75
     ------
      12290
     17206
     2458
     ------
     4.30150
          20
     ------
Feet 86.030
```

7. *Masons Work.*

Masons measure their work, sometimes by the foot solid, sometimes by the square foot or yard, and sometimes by the rod, that is, 21 feet long and 3 feet high, viz. 63 square feet. But the rod differs in different places.

Ex. 1. A wall of hewn stone is 24 feet 6 inches long, 9 feet 6 inches high, and 2 feet 3 inches thick: How many solid feet are in that wall?

```
                          F.  in.
        24.5              24   6
         9.5               9   6
      ------             --------
       1225              220   6
      2205                12   3
      ------             --------
      232.75             232   9
        2.25               2   3
     --------            --------
      116375             465   6
      46550               58   2   3
     46550               -----------
     --------            523   8   3
Feet 523.6875
```

Ex. 2. If a wall be 218 feet 9 inches long, and 12 feet 6 inches high, how many square feet, yards, and rods, are in it?

218.75

218.75	*F.* *in.*
12.5	218 9
109375	12 6
43750	2625
21875	109 4 6
9)2734.375 feet.	2734 4 6
7)303.819$\cancel{4}$ yards.	
43.402$\cancel{7}$ rods.	

8. *Paving.*

Paving and causeying are measured by the square yard, of 9 square feet.

Ex. 1. A paved walk is 92 feet 6 inches long, and 8 feet 4 inches broad: How many yards does it contain?

	F. *in.*
92.5	92 6
8.$\cancel{3}$	8 4
740.0	740
30.8$\cancel{3}$	30 10
9)770.8$\cancel{3}$	770 10
Yards 85.6,481,	

Ex. 2. A stable, 19 feet 6 inches long, and 12 feet 6 inches broad, is to be floored or causeyed with hard bricks, called *clinkers*, each 6 inches long, and 3 inches wide: How many clinkers will it require?

```
  F. in.   in.
  19 6 = 234
  12 6 =  150
        ------
        1170
        234
        ------
6 × 3 = 18)35100(1950 clinkers. Anſ.
        18···
        -----
        171
        162
        -----
          90
          90
          --
```

V. *Board and Timber Meaſure.*

1. *Board.*

To meaſure board or plank, is to find the ſuperficial content or area of a rectangle. See Prob. 2.

R U L E.

Multiply the length in feet by the breadth in inches, divide the product by 12, and the quot is ſquare feet, every remainder being one twelfth of a ſquare foot.

Or, Reduce the length to inches; then multiply by the inches in the breadth; divide the product by 144, and the quot is ſquare feet, the remainder being ſquare inches.

Ex. 1. In a plank 14 feet long, and 8 inches broad, how many ſquare feet?

14
8
12) 112 ($9\frac{4}{12} = 9\frac{1}{3}$ *Anſ.* *Sq. feet.*
108
4

Or thus:

14
12
168
8
144) 1344 ($9\frac{48}{144} = 9\frac{1}{3}$
1296
48

By Gunter's ſcale.

Extend the compaſſes from 12 to the breadth 8; and that extent, ſet the ſame way, will reach from the length 14 feet to $9\frac{1}{3}$.

By the ſliding rule.

Set the breadth 8 inches on the ſlip or ſlider to 12 on the rule, and againſt the length 14 feet on the rule you have on the ſlip $9\frac{1}{3}$.

Ex. 2. In a plank 18 feet 6 inches long, and 15 inches broad, how many ſquare feet?

18.5
15
925
185
12) 277.5 *Sq. ft.*
23.125 = $23\frac{1}{8}$ *Anſ.*

Or thus:

18 6
12
222
15
1110
222
144) 3330 ($23\frac{18}{144} = 23\frac{1}{8}$
288
450
432
18

The operation by Gunter's ſcale and the ſliding rule is the ſame as above.

If the two ends of a plank differ in breadth, add them, and take half their ſum for a mean breadth.

If there be a number of planks of the ſame dimenſions, meaſure one of them, and multiply the content by the number of planks.

Ex. 3. If a plank be 9 inches broad, how many inches in length will make a foot ſquare?

9)144(16 inches in length. *Anſ.*

Hence is conſtructed the following table, ſhewing how many inches in length make a ſquare foot, at any breadth not exceeding 40 inches.

B.	*L.*	*B.*	*L.*	*B.*	*L.*	*B.*	*L.*	*B.*	*L.*
1	144	9	16	17	8.47	25	5.76	33	4.36
2	72	10	14.4	18	8	26	5.54	34	4.23
3	48	11	13.09	19	7.58	27	5.3	35	4.01
4	36	12	12	20	7.2	28	5.14	36	4
5	28.8	13	11.07	21	6.85	29	4.96	37	3.89
6	24	14	10.28	22	6.54	30	4.8	38	3.79
7	20.57	15	9.6	23	6.26	31	4.64	39	3.69
8	18	16	9	24	6	32	4.5	40	3.6

The uſe of the table is obvious. Look for the breadth of the board or plank under B, and under L you have the inches in length that make a ſquare foot: Take this length in the compaſſes from a ſcale of inches, and run that extent along the board, from end to end, and you have the number of ſquare feet that board contains; or you may, by this means, cut off from that board any number of ſquare feet required.

MORE

MORE EXAMPLES.

Length. Feet.		Breadth. Inches.		Content. Sq. feet.
14	———	18	———	21
9	———	$17\frac{1}{2}$	———	$13\frac{1}{8}$
$11\frac{1}{4}$	———	$7\frac{3}{4}$	———	$7\frac{1}{4}$
$9\frac{3}{4}$	———	$13\frac{1}{4}$	———	$10\frac{3}{4}$
$8\frac{1}{4}$	———	22	———	$15\frac{1}{8}$
$14\frac{1}{2}$	———	20	———	$24\frac{1}{6}$

2. *Timber.*

To meaſure timber is to find the ſolid content of a cube, parallelopiped, priſm, or cylinder. See Prob. 23. 24. and 25.

Equal-ſquared Timber.

By equal-ſquared timber is meant, ſuch as is in the form of a parallelopiped, whoſe baſes are equal ſquares or rectangles; and its ſolidity is found by multiplying the area of the baſe by the length.

Ex. 1. In a piece of ſquared timber 18 feet long, the ſide of the ſquare baſe being 15 inches, how many ſolid feet?

15
15
75
15
225
18
1800
225
144)4050(28.125

F.	in.	
1	3	
1	3	
1	3	
	3	9
1	6	9
		9
14	0	9
		2
28	1	6

Anſ. 28 feet, and half a quarter.

Inſtead

Inſtead of dividing by 144, you may divide by the component parts 12 × 12. Or you may reduce the length to inches, and then divide the product by 1728, or by 12 × 12 × 12.

In working by feet and inches, inſtead of multiplying by 18, I multiply by the component parts 9 × 2.

By Gunter's ſcale.

Extend the compaſſes from 12 to 15 inches, the ſide of the ſquare; and that extent twice ſet will reach from the length 18 feet to the anſwer 28 feet, and ſomething more.

By the ſliding rule.

Set the length in feet on the ſlip to 12 on the girt line, and againſt the ſide of the ſquare 15 inches, on the girt line, you have the anſwer on the ſlip, 28 feet, and ſomewhat more.

Ex. 2. In a piece of ſquared timber 32 inches broad, 18 inches thick, and 14 feet 6 inches long, how many ſolid feet?

	F.	*in.*
32	2	8
18	1	6
256	2	8
32	1	4
576	4	0
14.5	14	6
2880	56	
2304	2	
576	58 feet. *Anſ.*	
144)8352.0(58 feet.		

The operation by Gunter's ſcale and the ſliding rule is the ſame as above, only the ſide of a ſquare equal to the area of the baſe muſt be found, by extracting the ſquare root of 576 = 32 × 18, viz. 24; or it may be found

found by dividing the extent between 18 and 32 on the ſcale into two equal parts, and the middle point will be 24, the ſide of the ſquare ſought But on the ſliding rule, ſet the lines C and D even at the ends; and againſt any ſquare number on C you have its ſquare root on D.

Some perſons, of inferior ſkill, add the depth and breadth together, and take half their ſum for the ſide of the ſquare. But this method is falſe; and if the depth and breadth differ widely, the error thence reſulting will be conſiderable. In the above example, $32 + 18 = 50$, and half of 50 is 25 for the ſide of the ſquare; and $25 \times 25 = 625$ for the area of the baſe; which multiplied into the length, would give a product by far too great.

Divide 1728, the cubical inches in a ſolid foot, by the area of the baſe in inches, and the quot will be the inches in length that make a ſolid foot.

The above rule is general, and extends to timber of all ſorts, that is of equal breadth and thickneſs from end to end, whether the baſe be ſquare, triangular, multangular, or round.

Hence is conſtructed the following table, which ſhews how many inches in length make a ſolid foot, the ſide of the ſquare, equal to the area of the baſe, not exceeding 30 inches.

B.	*Length.*	*B.*	*L.*	*B.*	*L.*	*B.*	*L.*	*B.*	*L.*
1	1728.	7	35.26	13	10.22	19	4.78	25	2.76
2	432.	8	27	14	8.81	20	4.32	26	2.55
3	192.	9	21.3	15	7.68	21	3.92	27	2.37
4	108.	10	17.28	16	6.75	22	3.57	28	2.2
5	69.12	11	14.28	17	5.94	23	3.26	29	2.05
6	48	12	12	18	5.3	24	3	30	1.92

The uſe of the table is plain. Seek the ſide of the ſquare, equal to the area of the baſe, under B; and under L you have the number of inches in length that make a ſolid foot. Take this length from a ſcale of inches, and

and with it run the piece of timber from end to end, and you have the number of ſolid feet it contains. By this means, too, you may cut off any number of ſolid feet deſired.

Unequal-ſquared timber.

By unequal-ſquared timber is meant, any piece of ſquared timber whoſe baſes are unequal, whether they be ſquares or rectangles; and ſuch are moſt timber trees, when hewn and brought to their ſquares, the root-end being generally groſſer than the other.

The uſual cuſtomary way of meaſuring ſuch timber is to take the ſquare or rectangle at the middle of the piece for a mean baſe, and then proceed as taught above.

Ex. 1. In a piece of ſquared timber 20 feet long, the ſide of the ſquare baſe at the greater end is 25 inches, and at the leſſer end 9 inches: How many feet of timber are in that tree?

$$\begin{array}{r} 25 \\ \underline{9} \\ 2)\underline{34} \\ 17 \end{array}$$

17 the ſide of the ſquare in the middle.

$$\begin{array}{r} 17 \\ \underline{17} \\ 119 \\ \underline{17} \\ 289 \\ \underline{20} \\ 144)5780(40.13 \end{array}$$

Feet. 40.13 *Anſ.*

Ex. 2. In a piece of timber 18 feet long, the baſe at the greater end being 32 inches by 20, and at the leſſer end 16 inches by 10: How many feet of timber?

```
   32        20        24
   16        10        15
   ——        ——        ——
2) 48     2) 30       120
   ——        ——       24
   24        15       ———
                      360 area of mean base.
                      18
                      ———
                      288
                      36
                      ———  Feet.
                144)6480(45 Ans.
```

This cuſtomary way is erroneous; and the greater the difference between the two baſes is, the greater will the error be. But, notwithſtanding this, univerſal cuſtom prevails, and purchaſers will accept of no other ſort of meaſure. I ſhall, however, here obſerve, that a piece of unequal ſquared timber is the fruſtum of a pyramid; and the content may be accurately computed by Prob. 26. The former example done in this way follows.

```
                                  320
     32         16                640
     20         10                160
    ———        ———               ————
    640        160            3)1120(373.3
    160                                18
    ———                             ——————
    384                             29866
    64                              37333
   ——————     Root.                ——————
   102400 (320                 144)6720.0(46.6
   9
   ———
62)124
   124
   ———
```

Anſ. 46⅔ feet.

Round timber having equal baſes.

The cuſtomary way of meaſuring ſuch timber is, to gird the tree about the middle with a ſmall cord, and

 then

then take the fourth part of this girt for the ſide of a ſquare, with which they proceed as in ſquared timber.

Ex. 1. If a tree be 72 inches in circumference, or girt, and 24 feet long, how many feet of timber are in it?

	F.	*in.*
A fourth of 72 is 18	1	6
18	1	6
144	1	6
18		9
Area of the baſe 324	2	3
24	24	
1296	48	
648 *F.*	6	
144)7776(54 *Anſ.*	54	

Ex. 2. If a piece of round timber be 54 inches in girt, and 22 feet 6 inches long, how many feet are contained in it?

	F.	*in.*	*p.*		
$\frac{1}{4}$ of 54 is 13.5	1	1	6		
13.5			1	1	6
675			6	9	0
405		1	1	6	
135	1	1	6		
182.25	1	3	2	3	
22.5				22	6
91125		7	7	1	6
36450	27	10	1	6	
36450 *F.*	28	5	8	7	6
144)4100.625(28.476 *Anſ.*					

This cuſtomary way is falſe and erroneous; for if the circumference of a circle be 1, the area will be .07958; now

now the fourth of 1 is .25, the ſquare whereof is .0625, which is conſiderably leſs than the true area .07958; and conſequently the ſolidity thus computed will be conſiderably leſs than the true content.

But this cuſtomary way, falſe as it is, being eſtabliſhed by long practice, is univerſally received, and merchants will not purchaſe timber at any other meaſure.

I ſhall here, however, ſhow how the exact content may be found, by working the former example in a way that will give a true anſwer.

```
  54             2916
  54           .07958
 ---           ------
 216            23328
270            14580
----          26244
2916         20412
             ---------
             232.05528

232.055
   22.5
-------                      Feet.
1160275      144)5221.2375(36.25
464110
464110       True content   36.25
---------    Falſe content  28.47
5221.2375                   -----
                    Error    7.78
```

Round timber having unequal baſes, called tapering timber.

The cuſtomary way of meaſuring tapering timber is, to divide the tree into parts of eight or ten feet long, and then take the girt at the middle of each part, by which, as taught above, is computed the ſolidity of each part, and their ſum is the ſolid content of the whole tree.

Ex. A tree 20 feet in length is divided into two parts, each 10 feet long, the middle girt of one of the parts being 64 inches, and of the other 36 inches: How many cubic feet of timber are in that tree?

$\frac{1}{4}$ of 64 is 16

```
  16
  16
  --
  96
 16
 ---
 256
  10
----
2560
 810
```

$\frac{1}{4}$ of 36 is 9

```
  9
  9
 --
 81
 10
---
810
```

```
144)3370(23.4 feet. Anf.
    288
    ---
     490
     432
     ---
      580
      576
      ---
        4
```

This way of dividing the tree into parts is far more accurate than working by the middle girt of the whole tree; and the ſhorter the parts are, the nearer the truth will the anſwer be.

But this way of meaſuring has been ſhown to be erroneous when the baſes are equal; and it is ſtill more ſo in timber that tapers; and the more tapering the tree is, the greater will the error be.

The truth is, a piece of tapering timber ought to be conſidered as the fruſtum of a cone, and the ſolidity computed by the directions given in Prob. 27.

MORE EXAMPLES.

Length. Feet.		$\frac{1}{4}$ *of the girt.* Inches.		*Content.* Cubic feet.
18	———	8	———	8
12	———	15	———	$18\frac{3}{4}$
22	———	$8\frac{1}{2}$	———	11
$27\frac{1}{2}$	———	19	———	68.9

VI.

VI. *Gauging.*

My design in this place is not to explain the principles or practice of gauging at any great length, but only to give the reader some general notion of that art, so as to enable him to find the content of any common cask or vessel, or of a cistern, or floor of malt.

PROB. I.

To find divisors, multipliers, and gauge-points, with their uses.

282 cubic inches make an ale-gallon.
231 cubic inches make a wine-gallon.
268.8 cubic inches make a corn-gallon.
2150.42 cubic inches make a corn or malt bushel.

The numbers, then, 282, 231, 268.8, 2150.42, are the divisors; and if 1 be divided by these, the quots thence arising will be the equivalent multipliers, as exhibited in the following table.

TABLE I. For right-lined figures.

Divisors.		*Multipliers.*	
282	A. G.	.003546	A. G.
231	W. G.	.004329	W. G.
268.8	C. G.	.0037202	C. G.
2150.42	M. B.	.000 6502	M. B.

If the cubic inches in any vessel be divided or multiplied by these divisors or multipliers, the result will be ale-gallons, wine-gallons, &c.

Again, if the superficial inches or area of any right-lined figure be divided or multiplied by these divisors or multipliers, the result will be the area in ale or wine gallons, &c.; that is, the number of ale or wine gallons contained in the same at 1 inch deep.

Ex.

Ex. Suppose a plain figure, in form of a rectangle, 232 inches in length, and 64 inches in breadth, what is the area in ale, wine gallons, &c.

$$232 \times 64 = 14848 \text{ area in inches.}$$

By division.

282	)14848(52.652	ale-gallons.
231	)14848(64.277	wine-gallons.
268.8	)14848(55.238	corn-gallons.
2150.42	)14848(6.904	malt-bushels.

By multiplication.

14848 × .003546	= 52.651	ale-gallons.
14848 × .004329	= 64.277	wine-gallons.
14848 × .0037202	= 55.238	corn-gallons.
14848 × .00046502	= 6.904	malt-bushels.

But for circular areas another set of divisors and multipliers must be obtained, in the following manner, viz. .785398 is the area of a circle whose diameter is 1; by which, if the numbers 282, 231, 268.8, 2150.42, be divided, the quots will be a set of divisors. And if .785398 be divided by the same numbers, viz. 282, 231, &c. the quots will be a set of multipliers, as in the following table.

TABLE II. For circular areas.

Divisors.		*Multipliers.*	
359.05	A. G.	.002785	A. G.
294.12	W. G.	.003399	W. G.
342.24	C. G.	.002922	C. G.
2738	M. B.	.00036523	M. B.

If the square of the diameter of any circle be divided or multiplied by these divisors or multipliers, the result will be the area in ale, wine, corn gallons, or malt-bushels;

ſhels; that is, the number of ale, wine gallons, &c. contained in the ſame at 1 inch deep.

Ex. Suppoſe the diameter of a circle to be 72 inches, what is the area in ale, wine gallons, &c.?

$72 \times 72 = 5184$ the ſquare of the diameter.

By diviſion.

359.05)5184(14.438 ale-gallons.
294.12)5184(17.625 wine-gallons.
342.24)5184(15.147 corn-gallons.
2738)5184(1.893 malt-buſhels.

By multiplication.

$5184 \times .002785 = 14.437$ ale-gallons.
$5184 \times .003399 = 17.620$ wine-gallons.
$5184 \times .002922 = 15.147$ corn-gallons.
$5184 \times .00036523 = 1.893$ malt-buſhels.

If the area in ale, wine gallons, &c. of an ellipſe, be required, multiply the greater and leſſer diameter together; and then divide or multiply their product by the diviſors or multipliers for circular areas; and the reſult will be the area of the ellipſe in ale, wine gallons, &c.

Gauge-points are the ſides of ſquares, or the diameters of circles, whoſe content, at 1 inch deep, is one gallon, one buſhel, &c. and conſequently are the ſquare roots of the diviſors exhibited in the two preceding tables. Theſe gauge-points are marked on the ſliding rule, for rendering the operation by that inſtrument more ſimple, and are as follows.

Gauge-points for ſquares.	*Gauge-points for circular areas.*
16.79 A. G.	18.95 A. G.
15.19 W. G.	17.15 W. G.
16.39 C. G.	18.5 C. G.
46.37 M. B.	52.32 M. B.

The

The way of uſing theſe gauge-points will appear in what follows.

PROB. II.

To find the content in ale, wine gallons, &c. of any parallelopiped.

RULE.

Find the ſolid content in inches, by multiplying the three dimenſions continually; and this product, divided or multiplied by the diviſors or multipliers in Table I. will give the content in ale, wine gallons, &c.

Or, Find the area of the baſe in ale, wine gallons, &c. by Prob. 1.; and then multiply that area by the height or depth.

Ex. Suppoſe a parallelopiped, trough or ciſtern, to be 81 inches long, 25 inches broad, and 26 inches deep: Required the content in ale, wine gallons, &c.

```
     81      282    )52650(186.7   ale-gallons.
     25      231    )52650(227.92  wine-gallons.
   ----      2150.42)52650( 24.48  malt-buſhels.
    405
   162
   ----                   Or thus:
Area2025
     26                282)2025(7.18
  -----
  12150   And 7.18 × 26 = 186.68 ale-gallons, &c.
  4050
  -----
  52650
```

By the ſliding rule.

In order to work by the ſliding rule, you muſt find the ſide of a ſquare equal to the area of the baſe, by extracting the ſquare root of 2025; or, by the rule, thus: Set the lines C and D even at the ends; and againſt any ſquare number on C, you have the root on D, which in this example is 45; and accordingly,

Set

Set the gauge-point 16.79 upon D, to the depth 26 upon C, and againſt the ſide of the ſquare 45 upon D you have on C 186.7, the content in ale-gallons.

The operation for wine-gallons, malt-buſhels, &c. is the ſame, only uſe the proper gauge-points.

PROB. III.

To find the content in ale, wine-gallons, &c. of a veſſel whoſe baſes are ſimilar and parallel, but unequal, being the fruſtum of a pyramid.

RULE.

Find the area of each baſe, and a mean proportional between them, and multiply the ſum of theſe three by one third of the depth or height, and the product is the content in cubic inches; which, divided or multiplied by the diviſors or multipliers in Table I. gives the content in ale, wine-gallons, &c.

Ex. Suppoſe a veſſel, whoſe baſes are rectangles, the length of the greater baſe being 100 inches, and its breadth 70 inches, the length of the leſſer baſe 80, and its breadth 56, and the depth of the veſſel 42 inches: Required the content in ale, wine-gallons, &c.

$$\left.\begin{array}{r} 100 \times 70 = 7000 \\ 80 \times 56 = 4480 \\ \text{Mean proportional } \underline{5600} \end{array}\right\} \begin{array}{c} 7000 \times 4480 = 31360000, \\ \text{whoſe ſquare root is } 5600. \end{array}$$

$$\begin{array}{rr} & 17080 \\ \text{A th}^{\text{d}} \text{ of the depth} & \underline{14} \\ & 6832 \\ & \underline{1708} \end{array}$$

282)239120(847.94 ale-gallons.

231)239120(1035.15 wine-gallons.

By the ſliding rule.

Set the gauge-point 16.79 upon D, to 14, one third of the depth on C; and againſt 130.7, the ſquare-root

of the triple mean baſe 17080, upon D, you have on C 847.94, the content in ale-gallons.

PROB. IV.

To find the content of a cylinder in ale, wine-gallons, &c.

RULE.

Multiply the ſquare of the diameter by the depth or height, and divide the product by the diviſors for circular areas in Table II.

Ex. Suppoſe the diameter of a cylinder to be 56.5 inches, and the height 96 inches: Required its content in ale, wine-gallons, &c.

$$56.5 \times 56.5 \times 96 = 306456$$

359.05)306456(853.52 ale-gallons.

294.12)306456(1041.94 wine-gallons.

2738)306456(111.92 malt-buſhels.

By the ſliding rule.

Set the proper gauge-point on D to the depth on C, and againſt the diameter on D you have the content on C.

PROB. V.

To find the content of a veſſel in the form of the fruſtum of a cone, in ale, wine gallons, &c.

RULE.

This may be done, as in Prob. 3. by multiplying the ſum of the areas of the two baſes and a mean proportional, by the third part of the depth; but more eaſily in the following manner.

To the product of the two diameters add one third part of the ſquare of their difference; that ſum is the ſquare of a mean diameter; which multiply by the height or depth, and the product, divided by the diviſors

fors in Table II. gives the content in ale, wine gallons, &c.

Ex. Suppofe the greater diameter be 56.5 inches, the leffer diameter 19 inches, and the height 62 inches: Required the content in ale, wine gallons, &c.

$$56.5 \times 19 = 1073.5 \text{ product of the diameters.}$$

```
       56.5 × 19 = 1073.5 product of the diameters.
       19
      ----
Diff. 37.5 × 37.5 = 1406.25 fquare of their difference.
  3)1406.25(468.75
          1073.5
        --------
         1542.25 fquare of a mean diameter.
              62
        --------
          308450
         925350
        --------
359.05)95619.50(266.31 ale-gallons.
294.12)95619.50(325.1  wine-gallons.
  2738)95619.50(34.92  malt-bufhels.
```

By the fliding rule.

Set the proper gauge-point on D to the depth on C; and againft 39.27, the mean diameter or fquare root of 1542.25 on D, you have the content on C.

Gaugers ufually inch all fuch veffels; that is, they compute the content at every inch deep, and enter thefe contents in a table; and when they come to furvey, they have only to take the depth, and by comparing the wet inches with the table, they have the content by infpection.

PROB. VI.

To find the drip or fall of a tun.

The drip or fall of a tun is when the veffel is fet a little reclining, for conveniency of cleanfing, there being a plug-hole at C for the wafh to run off at.

By this poſition it will happen, that the liquor will riſe to E on one ſide, when the bottom is but juſt covered at D on the other; and when the liquor riſes to B, the part A B G will be empty. Now, if the content of the empty part A B G be ſubtracted from the content of the whole veſſel in a perpendicular poſition, the remainder will be the content of the veſſel in this oblique poſition. It is alſo required to find how much liquor will cover the bottom, or fill the part D C E.

			Inches.
Suppoſe	the diameter	A B	= 48
	the diameter	D C	= 40
	the height	m n	= 60
	the diameter	G H	= 45.5
	the diameter	F E	= 42.1
	the perpendicular	A L	= 17.46
	the height	n o	= 15.53

The content of the whole tun in a perpendicular poſition, found by Prob. 5. is 324.2 ale-gallons.

To find the content of the part A B G, obſerve the following rule.

To the ſquare of the top-diameter A B 48, add one half of the product of the ſaid top-diameter A B and bottom diameter G H 45.5; multiply this ſum by the perpendicular A L 17.46; and divide the product by triple the diviſors in Table II. viz. by 1077.15 for ale, and by 882.36 for wine.

Square of A B 48 = 2304 and A B 48 × G H 45.5 = 2184
Half of 2184 = 1092

3396 × 17.46 = 58294.16

1077.15)58294.16(54.12 ale-gallons.

Tun's content	342.2	A. G.
Content of A B G	54.12	A. G.
Remains G B C D	288.08	A. G.

To

To find how much liquor will cover the bottom, or fill D C E, work as follows.

To the ſquare of the bottom-diameter D C 40, add half the product of the ſaid bottom-diameter and top-diameter F E 42.1; multiply this ſum by the perpendicular height n o 15.53, and divide the product by triple the diviſors in Table II. viz. by 1077.15 for A. G. and by 882.36 for W. G.

Square of D C 40 = 1600, and D C 40 × F E 42.1 = 1684
Half of 1684 = 842

$$2442 \times 15.53 = 37924.26$$

$$1077.15)37924.26(35.207 \text{ ale-gallons.}$$

If the fruſtum of a pyramid ſtand reclined, inſtead of the diviſors aſſigned above, you muſt uſe the triple of the diviſors in Table I. viz. 846 for ale, and 693 for wine, the reſt of the work being the ſame.

PROB. VII.

To gauge a copper with a riſing crown.

Take the dimenſions thus.

Extend a pack-thread over the middle from A to B.

Set one end of a rod in C and D, and move it forward and backward, till you find the ſhorteſt depth E C and G D; or, inſtead of a rod, you may hang a plummet.

Next, ſet the end of the rod on the top of the crown at o, and you have F o; which ſubtracted from E C, leaves m C the height of the crown.

E G is equal to C D, the diameter at the bottom of the crown, and the diameter m n may be meaſured.

Now, ſuppoſe the dimenſions thus taken to be as follows.

Inches.

Inches.		*Inches.*
E C = 47		A B = 99
F o = 42		C D = 64
m C = 5		m n = 65

Now, to find the content of the copper from the crown upwards, the depth being 42 inches, take the diameter in the middle of every four, or of every fix inches, (the more diameters you take the better), and infert thefe diameters in the fecond column of the following table. Then, by Prob. 1. find the areas of the circles anfwering to thefe diameters in ale-gallons, and infert them in the third column; and next multiply each of thefe areas by 6, and infert the products in the fourth column; and their fum will be the content of A B m n; that is, fo much as the copper will hold after the crown is covered.

The crown is to be confidered as the fruftum of a fphere, and its content by Prob. 29. will be found to be 28.75 ale-gallons. But the content of the crown may more readily, and with fufficient accuracy, be found as follows.

The diameter C D is 64, and the area to this diameter is 11.408, which multiply by half the height of the crown, viz. 2.5, and the product is 28.52 for the content of the crown.

The content of the part m n D C, confidered as the fruftum of a cone, will be found to be 57.935 gallons; from which fubtract the content of the crown 28.52, and the remainder is 29.415 gallons, the quantity of liquor that will juft cover the crown.

Parts

Parts of the depth.	*Diameters.*	*Areas in ale-gall.*	*Content of every 6 inches.*
6	95.3	25.2945	151.767
6	90.1	22.6095	135.657
6	85.0	20.1223	120.734
6	80	17.8246	106.947
6	75.2	15.7499	94.499
6	70.5	13.8426	83.056
6	66	12.1319	72.791
Sum	-	-	765.451
To juft cover the crown			29.415
Content of the copper			794.866

PROB. VIII.

To gauge a cafk.

The finding the content of cafks is a difficult part in gauging, the fhape of cafks being fo different, that not any one rule will ferve for all. Gaugers therefore commonly diftinguifh cafks into four forms or varieties, viz. 1. Such as refemble the middle fruftum of a fpheroid; 2. the middle fruftum of a parabolic fpindle; 3. the lower fruftums of two equal parabolic conoids; 4. the lower fruftums of two equal cones, as reprefented in fig. 1.

1. The utmoft line in the figure, which exhibits the ftaves as very much curved, or arching, reprefents a cafk of the firft form; and is confidered as the middle fruftum of a fpheroid, being the moft capacious fort of cafk, or what will hold more liquor than a cafk of any other form.

2. The curved line next to the utmoft line, reprefents a cafk of the fecond form; and is confidered as the middle fruftum of a parabolic fpindle, being lefs capacious than the fpheroid.

3. The third curved line reprefents a cafk of the third form,

form, which is considered as the lower frustums of two equal parabolic conoids abutting upon one common base; and is less capacious than any of the two former.

4. The innermost line, which is quite straight from bung to head, represents a cask of the fourth form, and is considered as the lower frustums of two equal cones abutting upon one common base, being of all sorts of casks the least capacious.

Various rules are assigned in books of gauging for ascertaining the contents of these casks; but the most easy and practical way is, to find such a mean diameter as will reduce the proposed cask to a cylinder; and this may be done pretty accurately as follows.

Multiply the difference of the bung and head diameter by .7 for the spheroid, by .65 for the spindle, by .6 for the conoids, and by .55 for the cones, and add the product to the head diameter, and the sum is the mean diameter.

Ex. Suppose the bung diameter be 32 inches, the head diameter 24 inches, and the length 40 inches; the content in each variety is required.

The difference between the bung and head diameter is 8, and

Mean diameter.

$8 \times .7 = 5.6$, and $24 + 5.6 = 29.6$ for the spheroid.
$8 \times .65 = 5.2$, and $24 + 5.2 = 29.2$ for the spindle.
$8 \times .6 = 4.8$, and $24 + 4.8 = 28.8$ for the conoids.
$8 \times .55 = 4.4$, and $24 + 4.4 = 28.4$ for the cones.

The content in ale-gallons, found by Prob. 4. follows.

A. G.

$29.6 \times 29.6 \times .002785 \times 40 = 97.6$ for the spheroid.
$29.2 \times 29.2 \times .002785 \times 40 = 94.98$ for the spindle.
$28.8 \times 28.8 \times .002785 \times 40 = 92.4$ for the conoids.
$28.4 \times 28.4 \times .002785 \times 40 = 89.85$ for the cones.

By the sliding rule.

Set the gauge-point upon D to the length of the cask upon

upon C; and againſt the mean diameter on D you have the content upon C.

It may not be improper to obſerve here, that however geometrical and accurate the rules for caſk-gauging may be in themſelves, yet the content found by them can only come near the truth, becauſe caſks have only ſome reſemblance, but do not anſwer exactly in ſhape, to the forms deſcribed above.

Hitherto we have conſidered caſks as quite full; it now remains to point out a method of finding how much liquor is in caſks that are not full. This is called *ullaging*; the quantity of liquor in the caſk being the ullage; which, ſubtracted from the content of the whole caſk, leaves the quantity that would fill the empty ſpace or vacuity.

Now, the caſk may either be ſuppoſed to lie with its axis parallel to the horizon, as repreſented in Pl. 4. fig. 2.; or it may ſtand upright, having its axis perpendicular to the horizon. The former of theſe caſes we ſhall conſider in the firſt place, and the latter afterwards. And in both caſes the rules ſhall be adapted to the ſpheroidal caſk; becauſe, in the practice of exciſe, moſt caſks are conſidered as ſuch.

It is uſual to find the ullage by means of a table of ſegments ſuited to a cylindrical caſk; but as every one may not be provided with ſuch a table, we ſhall here aſſign a rule for finding the ullage, and that pretty accurately, without any table, as follows.

RULE.

Divide the wet inches, taken at the bung, by the bung-diameter; and if the quot exceed .5, ſubtract .5 from it, and add one fourth part of the remainder to the former quot, and then multiply this ſum by the content of the whole caſk; the product is the ullage.

But if, upon dividing the wet inches by the bung-diameter, the quot be leſs than .5, ſubtract the quot from .5, and from the ſaid quot ſubtract one fourth part of the remainder, and then multiply this laſt remainder by the content of the whole caſk.

EXAMPLE.

		Inches.
Suppose the dimensions of the given cask to be -	Length	32.5
	Bung-diam.	31
	Wet -	21
	Dry -	10
	Content	75.37 A.G.

$$31)21.000000\left(\begin{array}{l}.677419\\ .5\end{array}\right.$$

$$4).177419\left(\begin{array}{l}.044355\\ .677419\end{array}\right.$$

$$.721774 \text{ sum.}$$

And $.721774 \times 75.37 = 54.40010638$ ale gallons.

Subtract the ullage 54.4 from 75.37, the content of the cask, and the remainder will be the quantity of liquor that would fill the vacuity. Or, find the content of the vacuity thus: Divide the dry inches by the bung-diameter, and then proceed in all respects as you did in finding the ullage.

If the cask stand on end, as represented in Pl. 4. fig. 3. the quantity of liquor it contains may be found by the following rules.

1. Find the diameter D F at the surface of the liquor, by laying two rulers on different sides of the cask, and at the height of the liquor, so as to be parallel; then measure the distance between the rulers, and from it subtract the thickness of the staves on both sides.

2. To twice the square of the diameter DF add the square of the head-diameter LM, multiply the sum by the wet inches, and divide the product by the triple of the divisors for circular parts, viz. by 1077.15 for ale-gallons, and by 882.36 for wine-gallons.

EXAMPLE.

		Inches.
Suppose the dimensions to be	Length B G -	32.5
	Bung-diameter H I	27
	Head-diameter LM	23
	Wet E G -	8.5
	Diameter D F -	26.14
	Content of the cask	59.95 A. G.

D F 26.14 × 26.14 × 2 = 1366.6
Add 23 × 23 - = 529

1895.6
Wet inches 8.5

94780
151648

1077.15)16112.60(14.95 A. G.
Cask's content - 59.95

Wanted to fill the cask 45

If you work with the dry inches, the result will be the quantity of liquor that would fill the vacuity; which, subtracted from the content of the whole cask, will leave the quantity of liquor in the cask.

PROB. IX.

To gauge malt.

RULE.

If the malt be in a cistern, or lie on a floor in a rectangular form, find the area of the base in bushels, by multiplying the length into the breadth, and dividing the product by 2150.42; then multiply this quot by the mean depth; and the product is the content in bushels.

If the base be circular or elliptical, divide the square of the diameter, or product of the two elliptical diameters, by 2738, and multiply the quot by the mean depth.

Ex. 1. There is a ciftern, whofe length is 84 inches, and breadth 54 inches, and the mean depth 43.6 inches: Required the content.

$84 \times 54 = 4536$ And $2150.42)4536.00(2.1093$
And $2.1093 \times 43.6 = 91.965$ bufhels.

Ex. 2. There is a malt-floor, whofe length is 245 inches, and the breadth 184 inches, and the mean depth 5.6 inches: Required the content.

$245 \times 184 = 45080$ And $2150.42)45080.00(20.963$
And $20.963 \times 5.6 = 117.3928$ bufhels.

By the fliding rule.

Set the depth on the line MD to the length on the line N upon the flider; and againft the breadth on the line A you have the content on the line marked B upon the flider.

F I N I S.

Direction to the BOOKBINDER.

Place the four copper-plates at the end of the volume, in fucceffive order; and few them fo as that every plate may, when opened out, be wholly without the book.

PLATE. I.

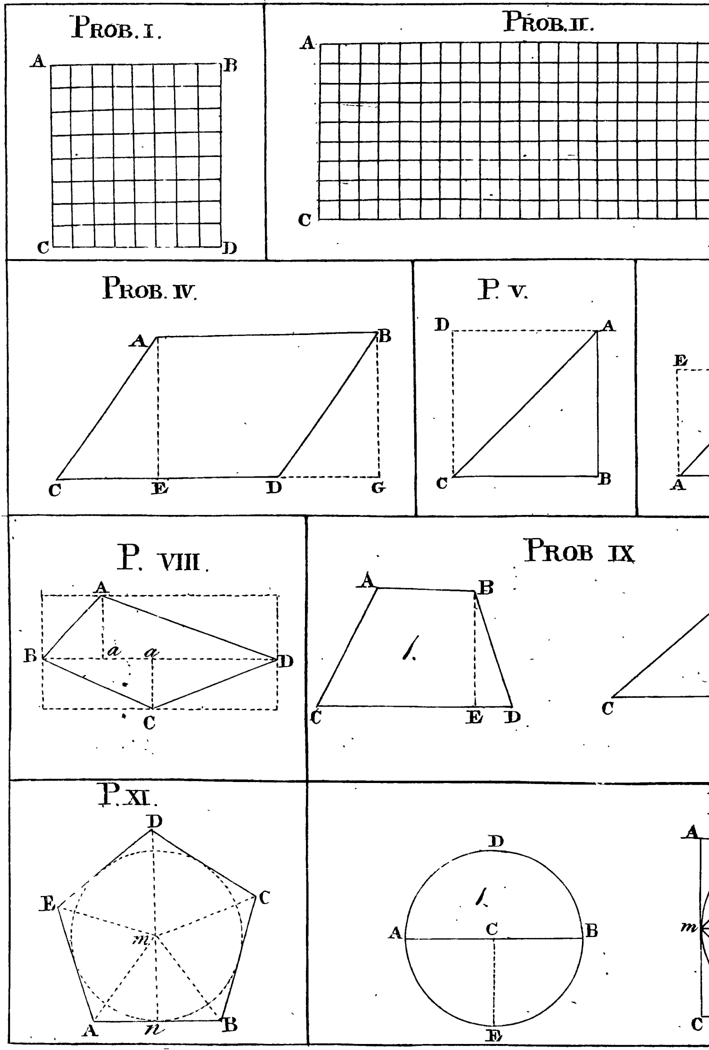
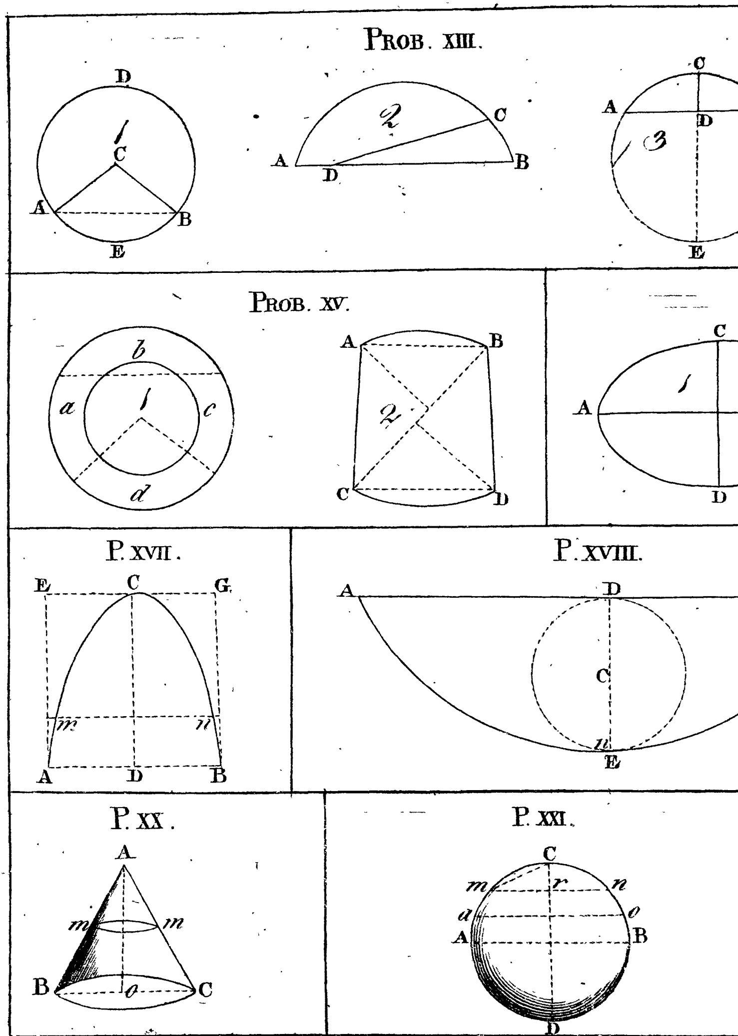


PLATE II.


PROB. XIII.
D
1
C
A
B
E
2
C
A
D
B
C
A
D
3
E
PROB. XV.
b
a
1
c
d
A
B
2
C
D
C
A
1
D
P. XVII.
E
C
G
m
n
A
D
B
P. XVIII.
A
D
C
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P. XX.
A
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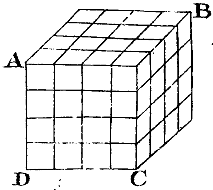
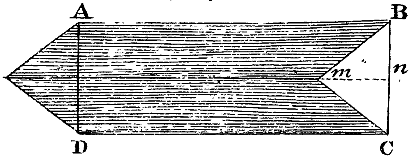
PLATE III

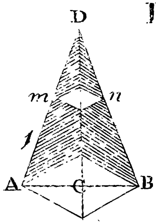
PROB. XXIII

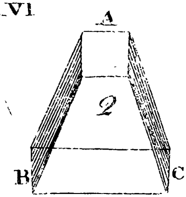


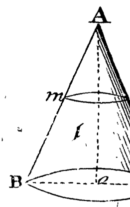
PROB. XXIV



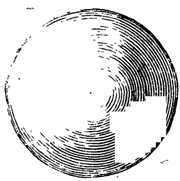
PROB. XXVI



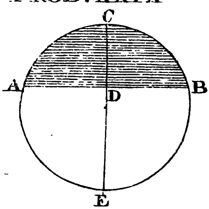




PROB. XXVIII

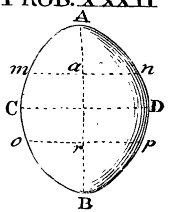


PROB. XXIX



PROB

PROB. XXXII



PROB. XXXIII

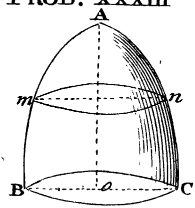
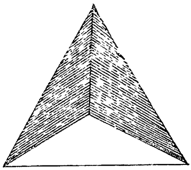


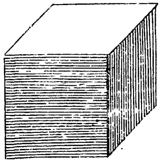
PLATE IV

PROB. XXXV.

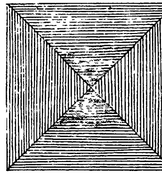
Tetraedron



Hexaedron



Octaedron



SURVEYIN

FIG. 1.

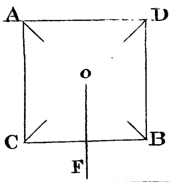
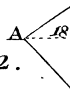
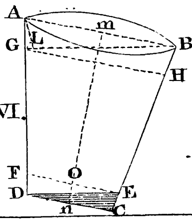


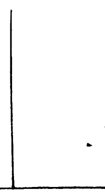
FIG. 2.



GAUGING

PROB. VI.





PROB. VIII.